A torque estimator-based control strategy for oil-well drill-string torsional vibrations active damping including an auto-tuning algorithm

Daniej Pavković, Joško Deur, Anton Lisac

1. Introduction

The boreholes of oil exploration and exploitation wells are typically drilled by means of a rock cutting tool (drill bit), which is attached at the end of a rather long drill string consisting of many smaller interconnected drill-pipe sections, and driven by a speed-controlled electrical drive (Beck et al., 1996). Due to large lengths and small cross-sections of the drilling pipes, low tool inertia, and emphasized tool vs. rock bed friction, the overall drill-string electrical drive is prone to poorly damped torsional vibrations including the stick–slip behavior (Jansen & van den Steen, 1995; Mihajlovic, Van Veggel, Van de Wouw, & Nijmeijer, 2004). These vibrations can be provoked either by the variable cutting/friction forces or the time-varying operator’s commands, such as a sudden change of drill-string servomotor speed reference or variations of the weight-on-bit (WoB) command (especially in the case of manual WoB control). Apart from the torsional drill-string vibrations, the drill string system is also subject to lateral and axial vibrations caused by drill string vs. borehole and tool vs. rock bed interactions (Christoforou & Yigit, 2003; Jansen, 1993).

Since the torsional vibrations contribute to aging and wear of drill-string drive components and reduce the efficiency/productivity of the drilling process, they need to be suppressed by means of appropriate vibration-damping measures. This can be achieved by designing a passive vibration absorber mounted at the bottom side of the drill string (Navarro-López & Suarez-Cortez, 2004; Vigué et al., 2009). Another hardware-based measure, proposed in Richardson and Küttel (2000), includes an additional hydraulic torque drive system placed within the bottom-hole-assembly (BHA). The main disadvantages of such hardware-based solutions are that they increase the drilling system complexity and cost, and they may need to be specifically tailored for the particular drilling structures (Tucker & Wang, 1999; Abdulgalil & Siguerdidjane, 2005; Al-Hiddabi, Samanta & Seibi, 2003; Puebla & Alvarez-Ramirez, 2008; Navarro-López & Liceaga-Castro, 2009; Serrarens, van de Molengraft, Kok, & van den Steen, 1998; Karkoub, Zribi, Elchaar, & Lamont, 2010), and (ii) indirect approaches typically based on a WoB control system coupled with the main drill-string speed controller (Canudas-de-Wit, Rubio & Corchero, 2008; Navarro-López & Suarez-Cortez, 2004).

The engineering practice has shown that the active damping control strategy should meet the following requirements:

1. Effective attenuation of drill-string torsional vibrations. The higher the level of vibration damping is, the drilling process...
becomes more productive and easier to handle by the operator.

2. Prevention of the so-called drill-string drive back-spinning, caused by a limited braking power of the motor power converter, drill string compliance, and the tool stick–slip friction effects, in order to avoid the potential damage to the drive (e.g., unfastening of drill string sections and gearbox damage).

3. Control strategy adaptation with respect to varying drill string length and variable (borehole-dependent) tool-side configuration, which can be realized by means of either auto-tuning or self-tuning approach (cf. Aström & Wittenmark, 1989). In order to facilitate reliable on-line tuning of the controller, a simple and straightforward analytical tuning procedure is desired.

4. Control strategy robustness with respect to variations of other drill-string drive parameters such as motor torque constant and drive moments of inertia.

5. Simplicity of implementation is also desired, so that the control strategy can be realized on a typical low-cost industrial programmable logic controller (PLC).

In terms of the requirement on simplicity of the overall drill-string control system, the direct active damping approaches are mostly used in applications and they are considered in this paper. While various active-damping control strategies have been proposed in the literature, they do not appear to consider or meet all of the above requirements. Tucker and Wang (1999) consider a proportional–integral (PI) speed controller, whose design is based on a spatially distributed drill string model. However, this low-order controller may have difficulty in achieving a full potential of torsional vibration damping, and its tuning procedure may not be convenient for auto-tuning purposes. A full-order state controller based on the two-mass elastic process model is proposed by Al-Hiddabi et al. (2003), but the “critical” aspect of robustness of full-order nonlinear state estimation in the presence of unknown stick–slip friction is not considered. The controller sensitivity with respect to parameter variations has been addressed in the literature by extending the controller with a nonlinear drill-string parameter estimator (Puebla & Alvarez-Ramirez, 2008), or by using an inherently robust controller such as a sliding-mode controller (Abdulgalil & Siguerdidjane, 2005; Navarro-Lopez & Licêega-Castro, 2009) or an $H_\infty$ design-based controller (Serrarens et al., 1998). However, the control/estimation approaches proposed by Puebla and Alvarez-Ramirez (2008), Abdulgalil and Siguerdidjane (2005), and Navarro-Lopez and Licêega-Castro (2009) again require a full-order state feedback or the full-order estimator, which may not be feasible/robust in practical applications. The input–output type $H_\infty$ controller (Serrarens et al., 1998) appears to be well-suited for practical applications, but it is not quite clear from the presented results if such a constant-parameter high-order controller can provide consistent quality of performance for a wide range of drill string lengths and configurations. None of the available papers discusses the drill-string back-spinning phenomenon caused by the limited servomotor braking power. It should be noted, though, that a method of handling a stuck tool situation by means of a state controller with a WoB feedforward action is presented in Navarro-Lopez (2009), but it does not take into account the braking power/torque saturation effect.

The main aim of this work has been to develop and implement a drill-string drive control strategy that meets all of above design requirements. The overall structure of the proposed control strategy is shown in Fig. 1, with definitions of symbols (variables) given in the Nomenclature section. Among several possible state controller structures (Deur, Koledić, & Perić, 1998; Leonhard, 1985; Schäfer, 1992), the PI motor speed controller extended with a drill-string torque feedback is used (Section 3), because of the following advantages: (i) good performance (fast response) for the particular case of low load/tool inertia (Deur et al., 1998) and (ii) no requirement for a tool speed estimator which is typically sensitive to time-varying nonlinear load friction effects (Schäfer, 1992). A Luenberger observer, which is also described in Section 3, is used to estimate the drill-string torque as a disturbance variable (Pavković, Perić, & Deur, 2000; Schäfer, 1992; Weilrich, 1978). The special case of pure PI controller (no

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$c$</td>
<td>drill-string stiffness coefficient (N m/rad)</td>
</tr>
<tr>
<td>$d$</td>
<td>drill-string damping coefficient (N m s/rad)</td>
</tr>
<tr>
<td>$d_{i,dp}$, $d_{b,dp}$, $d_{c}$</td>
<td>inner diameters of regular drill-pipes, HWDP and collars (m)</td>
</tr>
<tr>
<td>$d_{o,dp}$, $d_{b,do}$, $d_{oc}$</td>
<td>outer diameters of regular drill-pipes, HWDP and collars (m)</td>
</tr>
<tr>
<td>$D_2$, $D_3$, $D_4$</td>
<td>damping optimum characteristic ratios of closed-loop system</td>
</tr>
<tr>
<td>$D_{2s}$, $D_{3s}$</td>
<td>characteristic ratios of Luenberger estimator</td>
</tr>
<tr>
<td>$I_1$, $I_2$</td>
<td>motor-side and tool-side inertia (kg m²)</td>
</tr>
<tr>
<td>$l_{dp}$, $l_{hw}$, $l_c$</td>
<td>lengths of drill-pipes, HWDP and collars (m)</td>
</tr>
<tr>
<td>$G$</td>
<td>shear modulus of steel (N/m²)</td>
</tr>
<tr>
<td>$i$</td>
<td>gearbox (transmission) ratio</td>
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<tr>
<td>$K_{1e}$</td>
<td>Luenberger estimator speed gain (s⁻¹)</td>
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<tr>
<td>$K_{2e}$</td>
<td>Luenberger estimator torque gain (N m/rad)</td>
</tr>
<tr>
<td>$K_{3e}$</td>
<td>Luenberger estimator torque derivative gain (N m s⁻² rad⁻¹)</td>
</tr>
<tr>
<td>$K_R$</td>
<td>Pim and PI controller proportional gain (N m s/rad)</td>
</tr>
<tr>
<td>$K_m$</td>
<td>Pim controller torque gain</td>
</tr>
<tr>
<td>$T_z$</td>
<td>Pim and PI controller integral time constant (s)</td>
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<tr>
<td>$m_{1e}$</td>
<td>motor torque command (N m)</td>
</tr>
<tr>
<td>$m$, $m_{2r}$</td>
<td>drill-string torque and tool friction torque (N m)</td>
</tr>
<tr>
<td>$M_{\max,op}$</td>
<td>operator’s and actual (modified) torque limit (N m)</td>
</tr>
<tr>
<td>$M_{es}$</td>
<td>safe torque reserve (N m)</td>
</tr>
<tr>
<td>$M_1$, $M_2$, $M_3$, $M_4$</td>
<td>breakaway torque and Coulomb friction torque (N m)</td>
</tr>
<tr>
<td>$\beta_1$, $\beta_2$</td>
<td>inertia ratio and frequency ratio</td>
</tr>
<tr>
<td>$T$</td>
<td>sampling time (s)</td>
</tr>
<tr>
<td>$T_m$, $T_Z$</td>
<td>equivalent time constant of motor open-loop system (s)</td>
</tr>
<tr>
<td>$\Delta z$</td>
<td>torsional angle (rad)</td>
</tr>
<tr>
<td>$\Delta \omega$</td>
<td>speed difference (rad/s)</td>
</tr>
<tr>
<td>$\omega_1$, $\omega_2$</td>
<td>motor and tool speed (rad/s)</td>
</tr>
<tr>
<td>$\omega_{1s}$</td>
<td>motor speed at the torque saturation point (rad/s)</td>
</tr>
<tr>
<td>$\omega_{R,op}$</td>
<td>operator’s and actual (modified) speed reference (rad/s)</td>
</tr>
<tr>
<td>$\Omega_0$, $\Omega_{01}$, $\Omega_{02}$</td>
<td>drill-string drive, motor-side and tool-side natural frequencies (rad/s)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>damping coefficient of closed-loop control system</td>
</tr>
<tr>
<td>BHA</td>
<td>bottom-hole assembly</td>
</tr>
<tr>
<td>HWDP</td>
<td>heavy-weight drill-pipes</td>
</tr>
<tr>
<td>IAE</td>
<td>integral of absolute error</td>
</tr>
<tr>
<td>WoB</td>
<td>weight-on-bit (t)</td>
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load torque observer included; Deur & Perić, 1999) is considered, as well, and compared with the state controller with respect to performance and robustness features. The controllers and the observer are tuned according to the analytical damping optimum criterion (Naslin, 1968; Zäh & Brandenburg, 1987). The backspinning phenomenon is analyzed in detail, and an appropriate back-spinning prevention algorithm, acting upon the core feedback controller (Fig. 1), is proposed in Section 4. Adaptation of the overall control strategy is implemented in the form of an auto-tuning procedure (Section 5), which relies on the estimation of drill-string drive natural frequencies using the adaptive Kalman filter methodology (Gustafsson, 2000). The overall adaptive control strategy has been implemented on a low-cost PLC with floating-point arithmetic, and it has been experimentally verified on a hardware-in-the-loop (HIL) setup and in the field (on a commercial drilling rig). The description of the testing equipment and experimental results is given in Section 6. Section 7 includes the concluding remarks and discusses future research directions.

2. Drill-string drive model

This section presents the structure of a two-mass elastic model of the drill-string drive, including the tool friction sub-model.

2.1. Drive structure

A schematic representation of the drill-string drive is shown in Fig. 1. The drive comprises the following components (Beck et al., 1996; Jansen & van den Steen, 1995):

- Electrical servomotor equipped with a gearbox.
- Tool (drill bit) and drill collars (heavy, thick-walled pipes), which constitute the so-called bottom-hole assembly (BHA).
- Transition pipes or heavy-weight drill pipes (HWDP), which are used to make a transition between the stiff drill collars and flexible drill pipes (in order to avoid the fatigue-related issues near BHA).
- Regular drill pipes, with relatively thin pipe walls, which make up the major portion of the drill string.

2.2. Two-mass elastic model

The drill string is commonly modeled as a two-mass elastic system described by the block diagram in Fig. 2a (Leonhard, 1985), where the regular drill-pipes represent the torsional spring with the stiffness coefficient \( c \) and the damping coefficient \( d \).
\( c = \frac{G \pi d}{l_d} \left( d_i^4 - d_o^4 \right), \)
\( d = \frac{d_i^2 - d_o^2}{2}, \)

where \( G \) is the shear modulus for steel (\( G = 7.96 \times 10^6 \text{ N/m}^2 \)), while the geometrical parameters \( d_i, d_o, d_o, \) and \( l_d \) are defined in Fig. 1 (see also Nomenclature section), and \( d_i \) is the drill-pipe damping coefficient per unit length. Note that the damping term \( d \Omega_a \) may also include the damping effect of the so-called mud flow which removes the drilling debris from the bottom of the well.

The motor and gearbox inertias are lumped into the overall inertia at motor side \( J_f \). The load inertia \( J_l \) includes the tool inertia \( J_{tool} \) the collar inertia \( J_c \), the HWDP inertia \( J_{hw} \) and the regular drill-pipes inertia \( J_{dp} \) (Jansen, 1993):

\[
J_t = J_{tool} + J_c + J_{hw} + J_{dp} = J_{tool} + \rho \frac{\pi}{2} \left[ k(d_i^4 - d_o^4) + l_{hw}(d_i^2 - d_o^2) + \frac{l_d^3}{3}(d_i^2 - d_o^2) \right],
\]

where \( k(d_i^4 - d_o^4) \) and \( d_i^2 - d_o^2 \) are the inner and outer diameters of collars and HWDP, respectively.

The model in Fig. 2a can be conveniently transformed to the form shown in Fig. 2b, where the tool (load) quantities are referred to the motor shaft taking into account the gear ratio \( i \).

At mid-to-large drilling depths (e.g. \( l_d > 200 \text{ m} \)), the above system is prone to low-frequency torsional vibrations excited either by the motor action or by perturbations in the tool friction torque \( m_f \). From Fig. 2b, the characteristic natural frequencies of drill-string drive torsional vibrations are (Deur et al., 1998; Leonhard, 1985; Schäfer, 1992):

\[
\Omega_{01} = \sqrt{\frac{c}{J_f i^2}},
\]
\[
\Omega_{02} = \sqrt{\frac{c}{J_c i^2}},
\]
\[
\Omega_0 = \sqrt{\frac{c}{J_c i^2 + \frac{1}{l_d^2}}},
\]

where \( \Omega_0 \) is the natural frequency of free drill-string drive vibrations, \( \Omega_{01} \) is the natural frequency of motor vibrations (e.g. when the tool is stuck), and \( \Omega_{02} \) is the natural frequency of tool vibrations (e.g. when the motor is stiffly controlled). The respective damping ratios \( \zeta_0, \zeta_{01}, \) and \( \zeta_{02} \) of the above vibration modes are often neglected due to their small values (typically below 0.05).

The main natural frequency of drill-string vibrations \( \Omega_0 = \Omega_{01} \) typically lies in the range from 0.05 to 0.5 Hz depending on the drilling depth and drill string (tool) configuration (Jansen & van den Steen, 1995).

The two-mass elastic system behavior is also characterized by the so-called inertia ratio:

\[
r_M = \frac{l_d}{l_i} = \frac{\Omega_{02}^2}{\Omega_{01}^2},
\]

where small \( r_M \) values (i.e. \( r_M < 1 \)) correspond to the case of light load, which is critical from the standpoint of control (Deur & Perić, 1999; Leonhard, 1985; Schäfer, 1992), and characteristic for the drill-string drive.

Another characteristic dimension-less ratio (the so-called frequency ratio) is defined as

\[
r_M = \frac{\Omega_0 i}{T_m}.
\]

where \( T_m \) is the equivalent time constant of servomotor response dynamics given by

\[
G_m(s) = \frac{m_1(s)}{m_{1t}(s)} = \frac{1}{T_m s^{0.5} + 1}.
\]

with \( m_{1t} \) denoting the motor torque reference. The time constant \( T_m \) is typically around 10 ms which results in \( r_M < 1 \). This points out that the drill-string torsional compliance is the dominant dynamic effect.

### 2.3. Tool friction model

The tool friction torque \( m_f \) can be described by the generalized Striebeck static curve, which is shown in Fig. 3a and may be described by (Armstrong-Hélouvry, Dupont, & Canudas-de-Wit, 1994)

\[
m_f(\omega_2) = [M_C + (M_S - M_C) e^{-|\omega_2/\omega_0|/\delta} \arctan(\omega_2)],
\]

where \( M_C \) is the Coulomb or dry friction torque, \( M_S \) is the maximum static friction torque (breakaway torque), \( \omega_0 \), and \( \delta \) is the Striebeck speed, and \( \delta \) is the Striebeck coefficient. The friction parameters depend on a wide range of factors, such as the type of rock, the tool type, and the vertical force applied on the tool (so-called weight-on-bit, WOB). This static friction model appears to be well-suited for the analysis of dominant friction effects (e.g. stick–slip effect due to the friction drop \( M_P-M_C \) at the transition between stiction and sliding regimes), as suggested by Mihajlovic et al. (2004), and Navarro-López and Cortés (2007). The above model can be extended either in the sense of including more complex tool friction phenomena, such as tool vs. rock bed contact effects and BHA vs. borehole impacts (see e.g. Christoforou & Yigit, 2003; Ritto, Soize, & Sampaio, 2010), or by considering a static relationship between the main friction model parameters \( M_S \) and \( M_C \) and WOB parameter (see Navarro-López and Cortés, 2007) and adapting \( M_S \) and \( M_C \) in an open loop manner with respect to WOB. The latter approach is adopted in this paper, as it primarily deals with the drill-string rotational dynamics.

Friction in the zero-speed region is called static friction or stiction, and may take on any value from \( -M_S \) to \( M_C \). In order to avoid the problem of non-unique stiction description (Fig. 3a), and achieve high computing efficiency of the friction model, the Karnopp friction model (Fig. 3b, Armstrong-Hélouvry et al., 1994) is used in this work. The model has a variable structure, described by

\[
m_f(\omega_2) = \left\{ \begin{array}{ll}
[M_C + (M_S - M_C) e^{-|\omega_2/\omega_0|/\delta} \arctan(\omega_2)] & \text{for } |\omega_2| > \omega_0,
[M_C + (M_S - M_C) e^{-|\omega_2/\omega_0|/\delta} \arctan(\omega_2)] & \text{for } |\omega_2| \leq \omega_0,
\end{array} \right.
\]

where the sliding friction complies with Eq. (9), while the stiction corresponds to the applied torque to the friction element (the drill-string torque \( m \) in this particular case) saturated to the

![Fig. 3. Striebeck friction model (a) and Karnopp model (b).](image-url)
maximum stiction level $\pm M_s$ (the sign of breakaway torque $M_s$ corresponds to the sign of the drill-string torque $m$).

3. Active damping control strategy

A linear active damping control strategy based on the estimation of the drill-string torque is presented in this section. The control strategy tuning procedure is based on the damping optimum criterion (Naslin, 1968; Zäh & Brandenburg, 1987). The nonlinear friction action is not explicitly considered in the below design and analysis study. This is justified by the fact that the additive static+Coulomb friction term cannot destabilize the linear two-mass elastic system (Fig. 2) controlled by a motor speed controller, provided that the following conditions on the linear control system are satisfied (see the harmonic-balance nonlinear analysis and simulation results in Schäfer, 1992; Schäfer & Brandenburg, 1991):

(i) the system is stable for the constrained motion case (i.e. when the tool is stuck, $\omega_2 = 0$),
(ii) the system is tuned for an aperiodic response, which effectively prevents tool speed sign change after the breakaway friction drop $M_s \rightarrow M_o$ occurs, thus avoiding the risk of limit-cycle stick–slip motion.

Condition (i) is satisfied for the controller proposed in this paper (see below design method and Appendix A), while condition (ii) relates to the inherent goal of this work, which is the design of an active damping control system. Note that the limit-cycle stick–slip behavior could still be avoided even if the linear closed-loop system is not fully damped (condition (ii) is not fully satisfied), provided that the motor speed reference is sufficiently large (which is typically encountered during normal drilling).

3.1. Controller structure

The discrete-time controller structure is shown in Fig. 4a. The core proportional-integral (PI) controller can be extended by an additional drill-string torque feedback control loop based on the estimated drill-string torque referred to the motor shaft $\dot{m}_i$ (the so-called Plm speed controller (Deur et al., 1998; Pavković et al., 2000; Schäfer, 1992)). The torque command $m_1$ is supplied to the inner current (torque) control loop implemented within the power converter. Fig. 4b shows the equivalent continuous-time representation of the controller, where the effective time delay $T_{\text{ZOH}}$ and sampling process and speed reconstruction based on motor position differentiation (with sampling time $T=20$ ms) is lumped to the “parasitic” motor torque time constant $T_m$, thus giving the equivalent open-loop time constant $T_S = T_m + T$. In the presence of drill-string torque estimator dynamics, characterized by the equivalent time constant $T_{\text{eo}}$, the total equivalent open-loop time constant equals $T_S = T_m + T_{\text{eo}}$.

In the actual case of emphasized “soft coupling” ($r_{\text{EM}} < 0.1$, Section 2), the equivalent open-loop time constant $T_S < 1/\omega_0$ may be neglected. Assuming also negligible damping of torsional vibrations ($d = 0$, Section 2), the closed-loop control system given

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1 The Luenberger estimator can be tuned for relatively fast response without violating the requirement on favorable noise suppression ability (see Section 6).
by Figs. 2b and 4b can be described by the following transfer function:

\[ G(s) = \frac{\omega_2(s)}{\omega_r(s)} = \frac{1}{J_1 T_s^4 + J_2 T_s^2 + J_3 + 1} \]  

(11)

3.2. Controller tuning

The PI/PIm controller tuning procedure is based on the damping optimum criterion (Naslav, 1968; Zah & Brandenburg, 1987), which defines the closed-loop system characteristic polynomial as

\[ A(s) = D_0 s^4 + D_1 s^3 + D_2 s^2 + D_3 s + D_4 \]

(12)

where \( T_s \) is the equivalent time constant of the closed-loop system, and \( D_0, D_1, \ldots, D_4 \) are the characteristic ratios. In the optimal case \( D_1 = 0.5 \) (\( i = 2, \ldots, n \)), the closed-loop system without transfer function zeros has a quasi-aperiodic step response characterized by an overshoot of approximately 6% and the rise time of approximately 1.8 \( T_s \). In the case of a controller with reduced order \( r < n \), only the dominant characteristic ratios \( D_2, D_3, \ldots, D_4 \) are set to the optimal value of 0.5, which results in setting the dominant closed-loop poles near their optimal locations. In that case, the effect of non-dominant closed-loop poles (or non-dominant characteristic ratios \( D_0, D_1 \)) to the robustness of the closed-loop system needs to be analyzed separately.

By equating the characteristic polynomial in Eq. (11) with the additional dynamics

\[ \frac{d\hat{\omega}_m}{dt} = \frac{1}{J_1 (m_1 - \hat{m}_r) + K_3 (\omega_1 - \hat{\omega}_1)} \]

and the estimation error \( e = \omega_1 - \hat{\omega}_1 \) correction terms are included into each state equation (Schäfer, 1992):

\[ \frac{d\hat{\omega}_m}{dt} = \frac{1}{J_1 (m_1 - \hat{m}_r) + K_3 (\omega_1 - \hat{\omega}_1)} \]

with the motor torque given by \( m_1 \approx m_{1,g} \) due to the fast torque control loop dynamics (\( T_m < T_s \)).

Assuming \( d \approx 0 \) in Fig. 2b (Section 2), the tool speed referred to the motor shaft, \( \omega_2 \), can be then estimated as

\[ \hat{\omega}_2 = \omega_1 - \frac{1}{c} \hat{m}_r = \omega_1 - \frac{1}{J_1 \Omega_0} \hat{m}_r. \]

The tool speed estimate can be utilized in drill-string drive diagnostics, e.g. to check whether the tool is stuck (Section 4).

The Luenberger estimator is tuned according to the damping optimum procedure, as well. For that purpose, the estimator is conveniently represented by the transfer function between the estimated drill-string torque \( \hat{m}_r(s) \) and the actual (reconstructed) drill-string torque \( m_r(s) = m_r(s) = \hat{m}_r(s) - J_1 \omega_1(s) \):

\[ G_m(s) = \frac{\hat{m}_r(s)}{m_r(s)} = \frac{(K_{3,e} K_{3,e}^2 + 1)}{J_1 / K_{3,e}^2 s^3 + (J_1 + K_{3,e} s)^2 + (K_{2,e} K_{3,e}^2 + 1)} \]

The damping optimum tuning procedure (cf. Section 3.2) yields the following expressions for the individual Luenberger estimator gains:

\[ K_{3,e} = \frac{1}{D_{30} D_{30} T_{e_0}} \]

(21)

\[ K_{3,e} = \frac{J_1}{D_{30} D_{30} T_s} \]

(22)

\[ K_{3,e} = \frac{J_1}{D_{30} D_{30} T_s} \]

(23)

The characteristic ratios are set to optimal values (\( D_{30} = D_{30} = 0.5 \)) to provide a well-damped estimator response. The equivalent time constant is set to a relatively small value (\( T_{m_0} = 0.3 \) s in this work), i.e. the estimator dynamics should be much faster than the closed-loop control system dynamics.

The discrete-time implementation of the overall estimator is shown in Fig. 5, which is obtained by means of Z-transform assuming zero-order-hold elements (ZOH) at the estimator inputs \( m_{1,g} \) and \( \omega_1 \).

![Fig. 5. Block diagram of discrete-time Luenberger estimator of drill-string torque and tool speed referred to motor shaft.](image-url)
3.4. Control system damping and robustness analysis

The closed-loop response damping is analyzed with respect to the variation of inertia ratio \( r_M \) based on the root locus plots of the equivalent control system transfer function (11). The normalized root-locus plots in Fig. 6, which also include lines of constant closed-loop damping ratio \( \zeta \), indicate that the PIm controller can effectively damp the dominant pole pair for a large range of inertia ratios \( r_M \leq 0.3 \). The non-dominant poles of both control systems are relatively well-damped, with the PIm controller-based system prone to lower damping ratios at low inertia ratio values.

The robustness of the drill-string control system to modeling errors is analyzed algebraically (again by means of root-locus method) based on the closed-loop model composed of the process model in Fig. 2b (with \( m_{I2}=0 \) and \( d=0 \)), the controller in Fig. 4b (with \( T_S=0 \)), and the drill-string torque estimator given by the transfer function (20). The transfer function of the analyzed system is of the fourth order in the case of PI controller \( (K_m=0) \) and seventh order in the case of PIm controller. The drill-string resonant frequencies \( \Omega_0 \) and \( \Omega_{02} \) are assumed to be accurate (e.g. obtained by auto-tuning, see Section 5).

The results of analysis of the closed-loop damping in the presence of modeling errors are shown in Fig. 7. These results point out that the PI controller tuned for optimal damping \( (D_2=D_3=0.5) \) is fairly robust to modeling errors, except for very low values of the inertia ratio \( r_M \) (when the dominant closed-loop poles may become weakly damped according to Fig. 6). On the other hand, the PIm controller tuned for optimal behavior \( (D_2=D_3=D_4=0.5) \), may be sensitive to modeling errors \( (\varepsilon > 5\%, \varepsilon_m < -5\%) \) in a quite wide range of inertia ratios \( r_M \). However, Fig. 7 indicates that the robustness of the PIm controller can be significantly improved (without any major effect on the dominant dynamics) by increasing the non-dominant characteristic ratio \( D_4 \) from 0.5 to 0.9. Increase of the dominant characteristic ratio \( D_2 \) would further extend the PIm controller robustness margin. However, this makes the dominant pole pair less well-damped, i.e. it weakens the main advantage of PIm controller in terms of damping the dominant dynamics for a wide range of inertia ratios \( r_M \) (see the above discussion related to Fig. 6).

The closed-loop system robustness issues are further illustrated and additional insights are gained by means of open-loop system Bode diagrams shown in Fig. 7b for different controllers and tuning choices and for the case of inertia ratio \( r_M=0.25 \) and motor torque relative error \( \varepsilon_m=-0.1 \). According to the Bode diagram interpretation of the Nyquist stability criterion (Netushil, 1978), the closed-loop system is stable if and only if the difference between the number of positive and negative transitions \( N_p-N_n \) of the phase characteristic \( \phi(f) \) over the \( \phi=-180^\circ \) line, subject to \( L(f)>0 \), is equal to \( m/2 \), where \( m \) is the number of right-half-plane (RHP) poles of the open-loop transfer function \( G_0(s) \). For the case of PIm controller tuned with \( D_2=D_3=D_4=0.5 \), the internal fast drill-string torque feedback loop through the estimator introduces an RHP conjugate-complex pole pair \( (m=2) \) into the overall...
open-loop transfer function $G_o(s) = \omega_1(s)/[\omega_R(s) - \omega_1(s)]$. Thus, the above stability condition is not satisfied since $m/2 = 1 \neq N_p - N_m = 0$. Other closed-loop systems, whose open-loop transfer function poles are all located in the left-half plane ($m = 0$), are in fact all stable, because $N_p - N_m = m/2 = 0$ for $L(f) > 0$.

4. Drill-string back-spinning

This section presents an analysis of the so-called drill string back-spinning phenomenon, caused by the limited braking current (motor torque) of the power converter and stick-slip tool friction. Also, an appropriate extension of the drill-string control strategy for the back-spinning prevention is proposed.

4.1. Analysis of back-spinning phenomenon

The drill-string back-spinning occurs when the tool is stuck (a constrained motion case) and the motor torque becomes limited. This is a potentially hazardous situation and it can result in disruption of drill-string drive operation (e.g. due to drill-pipe unfastening or gearbox damage).

An example of the back-spinning phenomenon recorded in the field during drilling at the depth of 1300 m (see Appendix C, Borehole C) is shown in Fig. 8. The operator’s speed reference $\omega_{R,op}$ was around 1200 rpm. In the initial phase (constrained drive motion), the fast PI speed controller accelerates the motor towards the reference speed, but due to the stuck tool the motor torque $m_1$ (and also the drill-string torque $m/1$) is being slowly ramped up. After the motor has reached the torque limit for motoring operation ($t = 9 \text{ s}$), it starts to slow down until all of the motor momentum $J_1\omega_1$ is spent to further build up the drill-string torque $m/1$. After the motor speed falls to zero and becomes negative ($\omega_1 < 0$), the drill-string motor (i.e. power converter) switches to the braking mode, where the braking torque is suddenly limited due to limited braking power. Since the drill-string torque $m/1$ now far exceeds the reduced motor torque limit, a sudden increase of motor deceleration occurs, thus resulting in high peak values of negative motor speed (back-spinning interval).

After the drill string is unwound, and the motor speed increases towards and above zero, the drive acceleration period is repeated (a new back-spinning period starts). Since these undesired high-amplitude back-spinning speed oscillations are caused by the power electronics hardware limitation, they cannot be effectively suppressed by the active damping controller alone.

4.2. Back-spinning prevention logic

One way of facilitating release of the stuck tool would be by means of simultaneously manipulating the weight-on-bit (WoB) and the drilling servomotor speed target (as discussed in Navarro-López & Cortés, 2007). However, only the servomotor intervention is considered below, because WoB control is implemented (as in many present drilling rigs) by means of manual control of the hoisting system mechanical brake. Fig. 9 shows the block diagram representation of the proposed active damping control strategy extension for the prevention of back-spinning effect. The algorithm comprises a flip-flop logic (Fig. 9a) which detects if the tool is stuck (i.e. tool speed $\dot{\omega}_2 \approx 0$) and if a large torque reference $m_{1R}$ is commanded to the motor (close to the operator’s torque limit $m_{max,op}$). In that case, the flip-flop is set ($Q = 1$), and the speed reference is temporarily switched to a small negative value $\omega_{R,NEG} < 0$ in order to unwind the drill string in a controllable way. The speed reference $\omega_R$ is returned to the operator’s reference $\omega_{R,op}$ (the flip-flop is reset, $Q = 0$) when the drill string is sufficiently unwound ($m_{1R} = 0$), or if the tool is no longer stuck (relatively large estimated tool speed $\dot{\omega}_2$ is detected). Since the drill string should also be unwound when

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2 The power converter braking current (motor torque) is limited (typically to 15% of the nominal value) because the braking mode is not used under normal drilling operation. Hence, the braking-mode part of the power converter (a chopper) and the braking resistors are typically rated below the nominal motor current in the drilling operating mode.
the drive is being stopped, the small negative speed reference may also be temporarily applied when zero speed reference is commanded (Fig. 9a).

As shown in Fig. 8, the drill-string torque \( m_i \) may exceed the operator’s drilling torque limit \( M_{\text{max,op}} \). In order to avoid the potential deceleration of drive and entering the back-spooling cycle (Fig. 8), this torsional torque excess needs to be covered by the motor torque in terms of extending the operator’s torque limit \( M_{\text{max,op}} \). The extended motor torque limit should not be excessive in order to comply as closely as possible with the operator’s limit command. For that reason, the algorithm estimates the actual, speed-dependent drill-string torque excess (so-called safe torque reserve), and adds it to the operator’s torque limit when the back-spooling (BS) flag is set (Fig. 9b). The safe torque reserve is estimated according to the expression

\[
M_{\text{res}} = J_i \omega_{i1} \omega_{10},
\]

which is derived in Appendix A. The constrained-motion motor speed \( o \) obtained as (Appendix A)

\[
o = \frac{K_c}{K_c + J_i T_e (1 + 1/m_R)}.
\]

The total torque limit \( M_{\text{max}} \) is used within the controller saturation algorithm (Fig. 4), which also includes the so-called reset-integrator logic of the controller integral term saturation (Åström & Wittenmark, 1997).

5. Auto-tuning algorithm

The overall control strategy is tuned automatically based on the analytical tuning procedure in Section 3 and the basic drill-string data (Fig. 1) provided by the operator. However, for the sake of operator convenience, an auto-tuning procedure based on the on-line estimation of the drill-string parameters has been developed. As explained in Section 2, the regular drill pipes length \( l_{dp} \) and the BHA configuration affect the drill-string parameters. Therefore, the operator should periodically execute the auto-tuning algorithm to update the drive data, and re-calculate the optimal controller parameters. This may be done each time a new section of regular drill-pipes is added to the drill string, or even less frequently at larger drilling depths because of the inverse square-root dependence of resonance frequencies with respect to drill-pipe length \( l_{dp} \) (see Eqs. (1)-(5)).

5.1. Auto-tuning concept

According to the results from Section 3, the tuning of PI/Pim controller (Eqs. (13)–(16)) and Luenberger estimator (Eqs. (19), (21)–(23)), requires the values of drill-string resonance frequencies \( \Omega_a \) and \( \Omega_{02} \), as well as the drive inertia values \( J_1 \) and \( J_2 \).

The drill-string vibration modes with frequencies \( \Omega_a \) and \( \Omega_{02} \) can be excited temporarily, as illustrated in Fig. 10 by the experimental data obtained from the HIL setup described in Section 6. The natural frequencies \( \Omega_{02} \) and \( \Omega_a \) are then estimated on-line by applying the adaptive Kalman filtering methodology (Gustafsson, 2000).

In order to avoid nonlinear tool friction effects, the tool needs to be lifted from the bottom of the well during the auto-tuner execution. The drill-string parameter estimation includes the following characteristic steps (Fig. 10):

1. Estimation of tool vibrations natural frequency \( \Omega_{02} \). The drive is controlled by the fast PI speed controller tuned according to symmetrical optimum (Eq. (17)). This results in “stiff” control of motor speed \( \omega_a \) and the corresponding excitation of the tool resonant mode with natural frequency \( \Omega_{02} \), which is visible in the motor torque command \( m_{1R} \) (Fig. 10).

2. Estimation of natural frequency \( \Omega_a \). The motor torque is now held at a constant value (open-loop control), which corresponds to average motor torque from step 1. The drill-string free vibrations with the frequency \( \Omega_a \) are captured from the motor speed signal \( \omega_1 \) (Fig. 10). At the end of this stage, the remaining parameters \( \Omega_{01} \) and \( J_1 J_2 l_i^2 \) are calculated based on Eqs. (5) and (6), and the PI/Pim controller and the overall control strategy including the torque estimator and back-spooling prevention algorithm are tuned.

3. Validation step. The drill-string drive is controlled by using the active damping PI/Pim controller tuned at the end of stage 2, and the closed-loop motor speed response \( \omega_1 \) is compared to the first-order model behavior:

\[
\omega_1(t) = \omega_0 (1 - e^{-t/T_o}),
\]

where the equivalent time constant \( T_o \) is given by Eq. (13). The comparison is based on the integral of absolute error (IAE) performance criterion (\( k = \) sampling step):

\[
\mathcal{J} = \frac{1}{N} \sum_{k=0}^{N} |\omega_1(kT_o) - \omega_0(kT_o)|.
\]

If the performance index \( \mathcal{J} \) is lower than a predefined threshold value, the auto-tuner execution has been successful. Otherwise, a warning is issued, and the tuning can be based on the drill-string data input by the operator.

5.2. Parameter estimation

Assuming negligible drill-string damping, estimation of the natural frequencies \( \Omega_a \) and \( \Omega_{02} \) can be based on a simplified model of free oscillator:

\[
\frac{d^2x}{dt^2} + \Omega^2 x = 0,
\]

where \( x \) represents the observed drill-string drive variable (motor torque command \( m_0 \) in stage 1, and motor speed \( \omega_1 \) in stage 2), and \( \Omega \) is the natural frequency to be estimated. The discrete-time form of the oscillator model (27) is given by (\( T = \) sampling time):

\[
x(k) = 2x(k-1) - 1 + T^2 \Omega^2 x(k-2).
\]
Based on this model, the Kalman filter of the drive resonant frequencies is given by the following set of equations:

\[ P(k) = P(k-1) - K(k-1)H(k-1)P(k-1) + Q(k-1), \]  
\[ H(k) = -T^2x(k-2), \]  
\[ K(k) = \frac{P(k)H(k)}{P(k)H(k) + R(k)}, \]

where \( Q \) is the desired variance of estimated parameter perturbations, \( R \) is the variance of measurement noise, \( P \) is the variance of parameter estimation error, \( K \) is the estimator update gain, and \( \Omega \) is the estimated natural frequency. Note that the natural frequency squared \( \Omega^2 \) is estimated rather than the actual value \( \Omega \) in order to avoid linearization of the oscillator model (28).

In applications the measurement noise variance \( R \) can be set to unit value \( (R = 1) \), and the Kalman filter is tuned by means of desired variance \( Q \) of perturbations in the estimated parameter \( \Omega^2 \). In order to achieve a fast response in the initial phase of estimation (good tracking ability) the \( Q \) parameter is initially held at a relatively large value (e.g. for 3 s, large adaptation gain \( K \)), which is followed by \( Q \) parameter decrease (low adaptation gain \( K \)).

Since the oscillations of the motor torque command \( m_{1p} \) and the motor speed \( \omega_2 \) can also comprise a DC offset, these variables are first differentiated before feeding them to the adaptive Kalman filter. In order to avoid issues with possibly large noise levels, the adaptive Kalman filter inputs are low-pass filtered by a digital 6th-order Bessel filter (Cavicchi, 2000), with the cutoff frequency \( f_c = 0.7 \) Hz.

6. Experimental results

The active damping control strategy has been implemented and verified on the hardware-in-the-loop (HIL) experimental setup in preparation for the subsequent field tests.

6.1. Hardware-in-the-loop (HIL) setup

A drill-string hardware-in-the-loop (HIL) experimental setup has been developed in order to implement and validate the proposed control strategy under the laboratory conditions. The principal schematic of the drill-string HIL setup is shown in Fig. 11a. The setup consists of an induction motor (IM) equipped with an incremental encoder, which is used as a drill-string driving motor, and a permanent-magnet synchronous servomotor (PMSM) used as a loading machine (programmable load). The emulated drill-string torque signal is then used as a torque command for the loading servomotor (PMSM). Other drill-string model variables, such as the load speed \( \omega_2 \) and the friction torque \( m_{f2} \) are internal state variables used for later visualization and analysis purposes.

6.2. Results of HIL tests

Fig. 12 shows the comparative experimental responses of the conventional (stiff) PI controller, and active damping PI and PIm controllers with the associated drill-string torque estimator for the case of linear drill-string model (no tool friction) and 600 rpm step change of speed reference \( \omega_{1p} \). The system with the conventional PI controller is characterized by notable tool speed and motor torque oscillations. The PI controller tuned for active damping is characterized by slower (softer) response, and the amplitude of tool vibrations is significantly decreased. By applying the PIm controller, the tool vibrations are effectively suppressed, as it was predicted by the root locus analysis in Fig. 6.

The effectiveness of the proposed control strategy is further illustrated by the HIL experimental responses in Fig. 13 which emulate the case when the tool is located at the bottom of the well (drilling case, the tool friction model is included). As a consequence of drill-string compliance, light load inertia, and the static-Coulomb friction excess \( (M_e - 1.6 M_s) \), the conventional PI controller-based system exhibits stick–slip periodic motion with high speed and torque amplitudes (see Fig. 13 for \( t < 13 \) s, and cf. Jansen & van den Steen, 1995). After the speed control is switched from the conventional PI controller to the PIm controller, the torsional vibrations and related stick–slip motion at the tool side are quickly suppressed. The bottom plot in Fig. 13
also shows that the drill-string torque estimator is characterized by a favorable tracking ability and low noise levels.

The effectiveness of the proposed adaptive Kalman filter is illustrated by the $\Omega_0$ and $\Omega_{02}$ estimation results in Fig. 14 (based on the responses in Fig. 10). The proposed adaptive Kalman filter indeed results in fast response, and accurate steady-state values of the estimated natural frequencies (low levels of noise in the estimator steady-state).

The results of experimental examination of the back-spinning prevention algorithm are shown in Figs. 15 and 16. When the tool suddenly becomes stuck (emulated on the HIL setup by a sudden increase of tool friction $m_{2g}$), the drill string unwinding is quickly initiated (Fig. 15). The back-spinning prevention algorithm then periodically switches between the operator’s speed reference $o_{R,op}$ and a small negative internal speed reference (cf. Fig. 9a) which results in a controlled drill string winding and unwinding process that may be used to achieve the tool breakaway. After a few tool breakaway attempts, a zero speed reference is commanded by the operator and the drive is stopped with the control strategy safely unwinding the drill string. Fig. 16 shows the scenario where the tool suddenly breaks away (emulated on the HIL setup by a sudden decrease of tool friction). Such a scenario may occur in the field due to breaking of the rock formation at the bottom of the well, or because of the weight-on-bit (WoB) reduction. In that case, the control strategy with PIm controller effectively suppresses the resulting torsional and/or stick–slip vibrations caused by the sudden friction drop.

6.3. Results of field tests

The proposed active damping strategy has been tested on a commercial drilling rig shown in Fig. 17a, equipped with a 600 kW/3200 Nm top drive motor (Fig. 17b), and controlled by the same PLC hardware and software used in the HIL setup. Vertical drilling field tests were carried out on two different
locations in the Palmyra region, Syria, and were characterized by two different drilling depth ranges and drill string/BHA configurations (Boreholes A and B in Appendix C). The tests were first carried out at relatively large drilling depths (i.e. deeper than 3000 m, cf. Borehole A data). For that particular case the borehole was relatively narrow near the bottom of the well, which mandated relatively narrow bottom-hole assembly characterized by relatively small load inertia ($r_M = 0.15$). Under these light-load conditions the PI controller has shown more robust behavior than the $P_I$ controller (cf. robustness analysis in Section 3.4).

The active-damping PI controller has been compared with the default/commercial drill-string motor controller (a relatively fast PI controller). Fig. 18 shows the comparative test results in the case of sudden slowing down for the drill-string lifted from the bottom of the well. The active damping controller significantly improves the level of damping of torsional vibrations excited by the speed command change. The comparative results for the case of drilling with the stepwise WoB change from 6 to 8 tons are shown in Fig. 19. Again, the active damping controller is able to suppress the torsional vibrations much better compared to the default controller. Finally, the performance of the aforementioned controllers is compared based on the 48 hours monitoring of drilling torque $i_m$ and WoB by the on-site geological service, as shown in Fig. 20. The torque response in Fig. 20a indicates that the application of active damping controller results in much smoother drill-string operation compared to the default controller (the RMS value of drilling torque perturbations is reduced by more than 50%). Moreover, this performance improvement is obtained for the approximately 15% increase of the average WoB (Fig. 20b), thereby also improving the drill-string rate of penetration.

Additional field tests have been conducted for the case of relatively low drilling depths (approximately 1250 m), and larger tool outer diameter which is characterized by a higher inertia ratio $r_M = 0.2$ (cf. Borehole B data in Appendix C). The comparative $P_I$ vs. PI controller test results for several successive WoB increases are shown in Fig. 21. As predicted by the root locus analysis in Section 3, the $P_I$ controller provides a more effective...
damping of torsional vibrations, but at the same time it shows certain sensitivity to modeling errors (1.5 Hz oscillations appear in the motor torque command $m_{1R}$).

7. Conclusion

The paper has presented the design of an oil-well drill-string electrical drive speed controller aimed for active damping of drill-string torsional vibrations. The controller includes the proportional-integral (PI) action extended with a drill-string torque feedback (so-called PI$m$ controller). A Luenberger observer has been designed for the purpose of estimation of the drill-string torque and the tool speed. The controller and estimator parameters have been determined analytically by using the damping optimum criterion. An analysis of the relative stability (damping) and robustness of the overall control system has also been carried out. The analysis has shown that the PI$m$ controller provides more effective damping of the dominant torsional vibration mode when compared with the pure PI controller. However, the PI$m$ controller can become sensitive to the process modeling errors, particularly for the case of very low inertia ratio values (light tool inertia).

In order to avoid the potentially hazardous back-spinning of the compliant drill-string drive, which is caused by the limited braking power of the power converter and stick–slip tool friction, an appropriate back-spinning prevention algorithm has also been proposed. It continuously monitors the estimated tool speed and the motor torque, and safely unwinds the drill string if the tool is critically stuck. The controlled unwinding procedure is supported by the extended motor torque limit, which is made dependent on the reference motor speed in order to minimize the motor torque excess with respect to the operator’s torque limit command.

An auto-tuning algorithm has been proposed in order to adapt the overall control strategy with respect to varying drill-string length and tool configuration. The auto-tuner briefly excites the characteristic drill-string resonant modes, and estimates the corresponding natural frequencies required for the overall control strategy tuning. The natural frequency estimator is based on an adaptive Kalman filter, which includes simplified process dynamics described a free-oscillator model, and schedules its gain to speed up the estimator response.

The proposed adaptive control strategy has been implemented and initially verified on the developed hardware-in-the-loop (HIL) model of the drill-string drive, and finally field-tested on a commercial oil-drilling rig. The test results have indicated that the active-damping PI controller can provide superior torsional vibration suppression performance when compared to the traditional, fast PI controller. According to the long run borehole data, the active damping results in more than 50% decrease of drilling torque perturbations, and at the same time it improves the drill-string rate of penetration. The PI$m$ controller has shown a significant additional potential of torsional vibration damping, but it turned out to be sensitive to drill-string drive modeling errors for very low tool inertia configurations at large drilling depths. The experimental tests have also demonstrated good performance of the back-spinning prevention algorithm and the control strategy auto-tuning procedure.

The future research is going to be directed towards more detailed modeling of the tool vs. rock bed and the bottom-hole assembly vs. side-wall friction effects, and the development of an auto-drilling control strategy including weight-on-bit control.

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Appendix A. Estimation of motor torque reserve for back-spinning prevention

Fig. 22 depicts the drill-string responses prior to the occurrence of the back-spinning phenomenon (cf. Fig. 8). In the case when the tool becomes stuck (\(\omega_2 = 0 \Rightarrow \omega_2 = \text{constant}\)), increase of the motor position \(\Delta z = z_1 - z_2\). During the acceleration period and constrained motion interval (Fig. 8), the speed-controlled motor winds up the drill-string torsional “spring”, whereby the motor torque command closely matches the drill-string torque. When the motor torque becomes limited to the operator’s torque limit \(M_{\text{max(op)}}\), the remaining kinetic energy of the motor is given by

\[
\Delta E_{\text{kin}} = \frac{1}{2} J_1 \omega_1^2;
\]

where \(\omega_1\) is the motor speed at the saturation point. Assuming negligible motor friction losses, this kinetic energy is then transferred into the following increase of the potential energy of the drill-string torsional “spring” during the motor slowing down to zero:

\[
\Delta E_{\text{pot}} = \frac{1}{2} c \left( \Delta z_f^2 - \Delta z_s^2 \right),
\]

where \(\Delta z_f\) is the torsional angle at the saturation point and \(\Delta z_s\) is the torsional angle reached when motor speed drops to zero (Fig. 22).

This increase in potential energy is manifested as the increase of drill-string torque during drive slowing down to zero (Fig. 22):

\[
\Delta m = \frac{c}{J} (\Delta z_f - \Delta z_s).
\]

Provided that this additional torque (a torque reserve) is available to the speed controller, it would be possible to safely stop the motor by applying the zero speed reference, or unwind the drill string by applying a small negative speed reference, i.e. the speed control capability would be preserved.

Equating expressions (34) and (35), and inserting \(c/\Omega_1^2 = J_1 J_0\) (based on Eq. (3)), yields

\[
\sqrt{\Delta z_f^2 - \Delta z_s^2} = \frac{\alpha_1}{\Omega_1} \Delta \omega.
\]

Next, by taking into account that the following inequality always holds

\[
\Delta z_f - \Delta z_s \leq \sqrt{\Delta z_f^2 - \Delta z_s^2}
\]

and combining it with expressions (36) and (37), the following upper bound of the drill-string torque excess \(\Delta m\) is obtained:

\[
\Delta m \lesssim J_0 \Omega_1 \alpha_1.
\]

Thus, the safe torque reserve equals \(J_0 \Omega_1 \alpha_1\), as used in the back-spinning prevention algorithm in Fig. 9.

During the drill-string drive constrained motion (\(\omega_2 = 0\)), the motor speed closed-loop dynamics are given by the following second-order transfer function model, obtained from Figs. 2b and 4b for \(\omega_2 = 0\), \(x = 0\), \(\Omega_1 = 0\) and \(\Omega_2 = 0\):

\[
G_{\omega_2(\Omega)} = \frac{\omega_2(\Omega)}{\omega_2(\Omega)} = \frac{K_m}{J_1 T_s^2 + K_m T_s + K_m + J_1 \Omega_1^2 T_1 (1 + J_m)},
\]

which results in the DC gain given by Eq. (24) and used in the back-spinning algorithm in Fig. 9. Taking into account that \(\Omega_0 = \Omega_1 (1 + \lambda_2)\) (cf. Eqs. (5) and (6)), the damping ratio \(\zeta_c\) of the above second-order closed-loop model reads

\[
\zeta_c = \frac{K_m}{\sqrt{J_1 [K_m (1 + \lambda_2) + J_1 \Omega_1^2 T_1 (1 + J_m)]}}
\]

Fig. 23 shows the corresponding damping ratio vs. inertia ratio plots of the PI and PIm controller-based closed-loop systems. These plots point out that the control system has a well-damped behavior (\(\zeta_c > 0.71\)) in the case of constrained motion.

Appendix B. Downsizing of drill-string drive to HIL setup

The HIL model corresponds to a downscaled representation of the commercial drill-string drive characterized with the following parameters: \(J_1 = 10\) kg m\(^2\), \(J_2 = 432\) kg m\(^2\), \(i = 12\), \(c = 525\) N m/rad and \(d = 20\) N m/s (\(\tau_m = 0.3\), \(\tau_0 = \Omega_0/2\pi = 0.2\) Hz and \(\zeta = 0.025\)). In order to facilitate equal starting dynamics of the HIL-based induction motor (driving motor, subscript IM) and the drilling rig motor, the following relationship between the HIL driving motor inertia and drilling rig motor inertia needs to be satisfied (see e.g. Leonhard, 1985):

\[
J_{\text{IM}} = J_1, \quad \lambda = \frac{n_h}{n_{\text{IM}}}, \quad \frac{M_{\text{IM}}}{M_{\text{IM}}} = \frac{M_{\text{IM}}}{M_{\text{IM}}}
\]

For the particular drilling rig motor with \(n_h = 1800\) rpm and \(M_{\text{IM}} = 3000\) N m, and the chosen HIL induction motor with \(n_{\text{IM}} = 1390\) rpm and \(M_{\text{IM}} = 10.3\) N m, the required total inertia at the HIL motor shaft is \(J_{\text{IM}} = 0.0417\) kg m\(^2\). This inertia includes the actual induction motor inertia (\(J_m = 0.0045\) kg m\(^2\)), the fly-wheel disk inertia (\(J_{\text{FW}} = 0.0371\) kg m\(^2\), see Fig. 11a) and the loading servomotor inertia referred to the motor shaft (\(J_{\text{SM}} = 0.0001\) kg m\(^2\)). Similarly, the parameters of the linear part

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**Fig. 22.** Illustration of drill-string drive response prior to back-spinning.

**Fig. 23.** Damping ratio vs. inertia ratio plots for PI and PIm controller-based systems in constrained motion regime (\(\Gamma_0 = 0.2\) Hz, \(J_1 = 10\) kg m\(^2\)).
of the emulated drill-string are: 

\[ c_H = \frac{27}{2} \times 10^{-2} \text{ N m/rad}, \] 

\[ d_H = \frac{5.8 \times 10^{-4}}{r_{\text{dr}}}, \] 

and 

\[ J_{\text{dr}} = r_{\text{dr}} \times \text{m}_\text{dr} = 0.0125 \text{ kg m}^2. \] 

Finally, the Coulomb friction torque \( M_{\text{Coul}} = 2.1 \text{ N m} \) and breakaway torque \( M_{\text{BSF}} = 3.35 \text{ N m} \), which are used in HIL emulation of the drill-string drive, are scaled down from assumed values \( M_S = 12500 \text{ N m} \) and \( M_L = 7800 \text{ N m} \) by the torque ratio \( M_{\text{BSF},\text{Eng}} / M_{\text{Eng},\text{BSF}} = 310.7 \) and the gearbox transmission ratio \( (i = 12) \).

### Appendix C. Drill-string data

**Borehole A**

Drill-pipes: \( d_{\text{dr}} = 108 \text{ mm}, d_{\text{sp}} = 127 \text{ mm}, l_{\text{b}} = 890-970 \text{ m} \).

HWDP: \( d_{\text{hw}} = 76.2 \text{ mm}, d_{\text{bw}} = 127 \text{ mm}, l_{\text{b}} = 110 \text{ m} \).

Drill collars: \( d_{\text{cc}} = 71.5 \text{ mm}, d_{\text{wc}} = 209.6 \text{ mm}, l_{\text{c}} = 148 \text{ m} \).

**Borehole B**

Drill-pipes: \( d_{\text{dr}} = 108 \text{ mm}, d_{\text{sp}} = 127 \text{ mm}, l_{\text{b}} = 950 \text{ m} \).

HWDP: \( d_{\text{hw}} = 76.2 \text{ mm}, d_{\text{bw}} = 127 \text{ mm}, l_{\text{b}} = 150 \text{ m} \).

Drill collars: \( d_{\text{cc}} = 76.2 \text{ mm}, d_{\text{wc}} = 241.4 \text{ mm}, l_{\text{c}} = 210 \text{ m} \).

**Borehole C**

Drill-pipes: \( d_{\text{dr}} = 108 \text{ mm}, d_{\text{sp}} = 127 \text{ mm}, l_{\text{b}} = 210 \text{ m} \).

HWDP: \( d_{\text{hw}} = 76.2 \text{ mm}, d_{\text{bw}} = 127 \text{ mm}, l_{\text{b}} = 950 \text{ m} \).

Drill collars: \( d_{\text{cc}} = 76.2 \text{ mm}, d_{\text{wc}} = 241.4 \text{ mm}, l_{\text{c}} = 210 \text{ m} \).

### References


