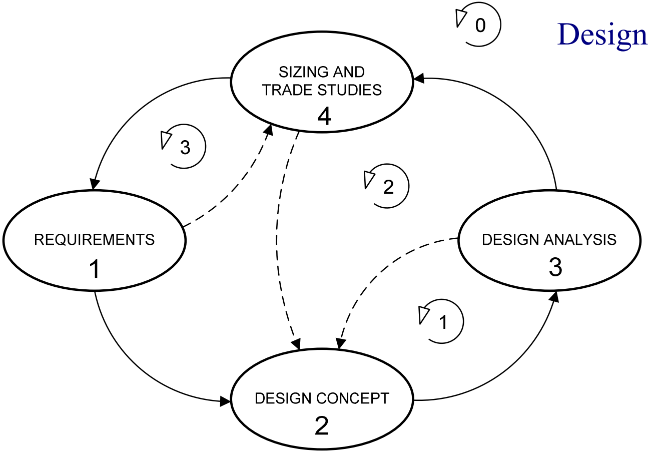
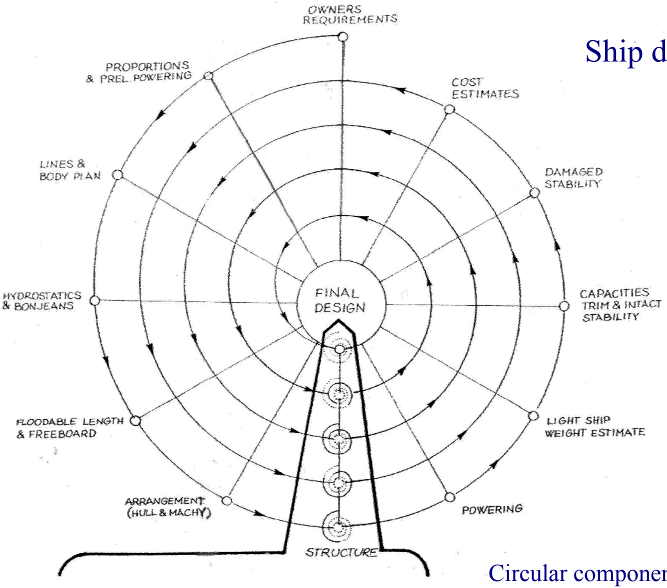
LECTURE 1b *SYNTHESIS MODEL*

Optimization methods and solution strategies

Design wheel

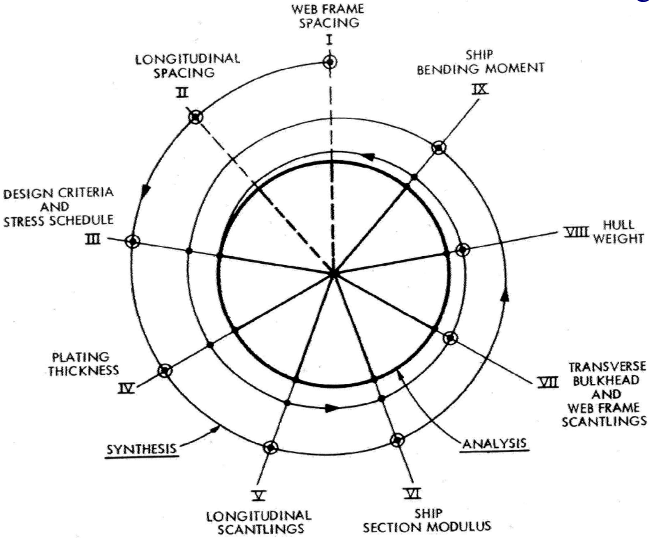


Ship design spiral



Circular component:
Radial component: accuracy level

Structure design spiral



Structural design parameters include

- scantlings,
- material (e.g. layer characteristics and orientation in composites),
- topology and geometry of the structure.

Since the latter two are usually fixed, except in shape optimization problems, the first two are commonly taken as free design variables.

Design quality measures are defined by using a set of design criteria functions (mappings) for :

- structural cost,
- weight
- safety evaluations.

Principles of design would require that for a good design:

- (Axiom I), the qualities, should be as much as possible uncoupled with respect to parameters,
- (Axiom II) the information content describing good design should be minimal (simplicity).

‘Best’ design(s) can be determined by **three classical ways of decision making**

- **lexicographical ordering** of priorities (method selects among the ‘best’ candidates regarding the first priority those that are the ‘best’ regarding second priority, etc).;
- **goal seeking** (construction of metric or ‘distance’ measure to the target design);
- construction of **value function** (combination of attribute functions as ultimate quality measure).

Description of sets/spaces and transformations

Figs. 1a-d give visual insight into concepts encountered in the realistic DS formulations (dominance, fuzziness, metrics).

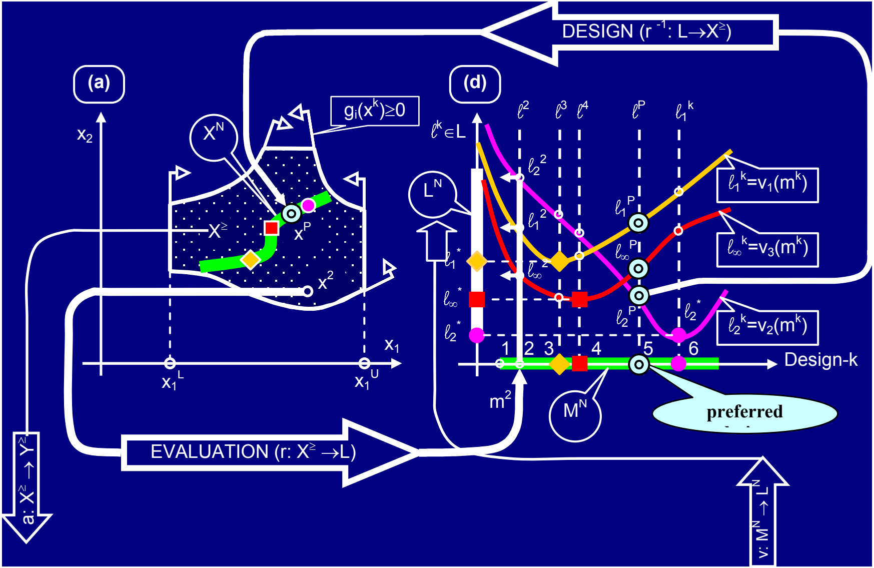
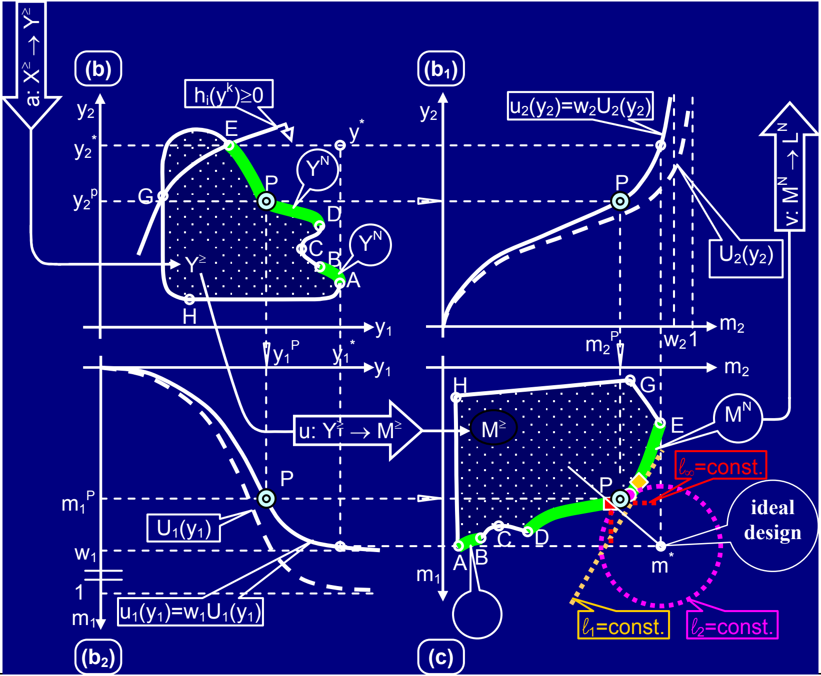

DESIGN (r -1: L→X≥)
(a)
(d)
x2
x1
XN
gi(xk)≥0
X≥
xP
x2
x1L
x1U
ℓk∈L
ℓ2
ℓ3
ℓ4
ℓP
ℓ1k
LN
ℓ1k=v1(mk)
ℓ∞k=v3(mk)
ℓ2k=v2(mk)
1 2 3 4 5 6
Design-k
m2
MN
preferred
EVALUATION (r: X≥ →L)
a: X≥ → Y≥
v: MN → LN

Figure 1

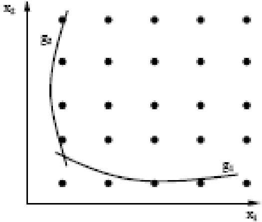

a: X≥ → Y≥
(b)
(b1)
(b2)
(c)
hi(yk)≥0
y2
y2*
y2p
y*
E
G
P
D
C
B
A
H
YN
Y≥
y1
y1P
y1*
u2(y2)=w2U2(y2)
U2(y2)
m2
m2P
w2
1
v: MN → LN
u: Y≥ → M≥
M≥
MN
m1
m1P
w1
U1(y1)
u1(y1)=w1U1(y1)
ℓ∞=const.
ℓ1=const.
ℓ2=const.
ideal design
m*

Design space

- *Design space **X***- is spanned by the free design variables x_i, i=1,.., nv. (*see Fig. 1a)*

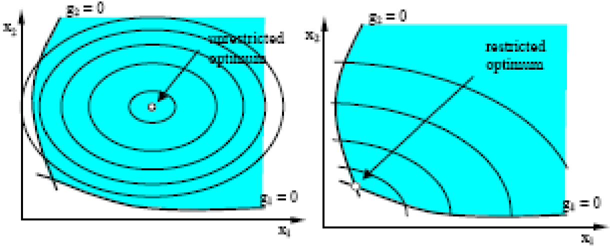
- Each design k is represented as a point $\mathbf{x}^k=\{x_i\}$ (e.g. $\mathbf{x}^2$ or $\mathbf{x}^P$) in this space.

- Some of the variables are discrete (no. of stiffeners, standard profiles). That could strongly influence problem formulation.



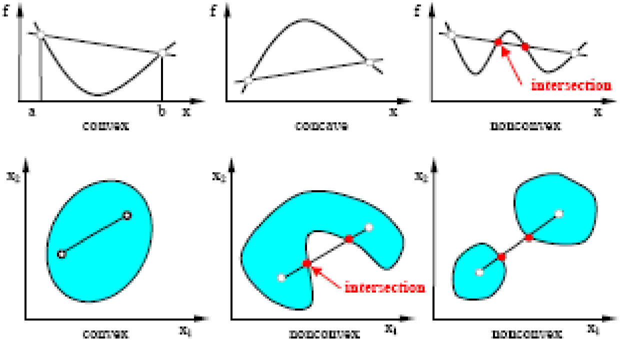
Design space

- Designs in subspace of feasible designs $\mathbf{X}^{\geq}$ satisfy:
  - failure criteria $g_i(\mathbf{x}) \geq 0$
  - production requirements,
  - functionality requirements,
  - min-max bounds x_i^L and x_i^U
  - other constraints.



Design space

- Note that $\mathbf{X}^{\geq}$ may be convex (line connecting two designs lies in $\mathbf{X}^{\geq}$) or non-convex as in Fig. 1a.



- Note also that it can be multiply connected (containing holes for e.g. resonance avoidance in vibration problems) and

Dimensionality

- *Dimensionality* of the problem is given by the number of variables nv.
- For concept structural design nv~40.
- Preliminary/Initial design would require nv~200/1000.
- The **"curse of dimensionality"**. Most of design variables are structural scantlings and spacing of girders on 2D (midship section, bulkheads) or 3D structures.
- Dimensionality is higher (100-1000) only in few works. Constraints $g_i(.) \geq 0$ are either global and local strength formulae or Rules. Structural response is calculated using FEM or analytical methods.

Attribute space

*Attribute space **Y** - (see Fig.1b)* is spanned by design attributes y_i.

- The mappings $\mathbf{y}^k\mathbf{=a(x}^k\mathbf{)}$ or $\mathbf{a} : \mathbf{X}^{\geq} \rightarrow \mathbf{Y}^{\geq}$ are used to form the attribute space (or outcome space) $\mathbf{Y}^{\geq}=\{\mathbf{y}^k\}$.
- For each feasible design $\mathbf{x}^k$ in **X** the design quality measures (attribute values) $\mathbf{y}^k=\{y_i\}$ define its corresponding point in **Y** space.
- Note that several points (designs) in **X** may map into a single point in **Y** (same weight, cost, etc.). In addition, x_i and y_j values are not mutually comparable and have **different units**

Attribute and Design spaces

- **X** and **Y** are **not metric spaces** i.e. there is no distance measure among designs.
- The **comparison of designs** is possible only within single variable x_i or attribute y_j.
- If **direction is selected** for the quality improvement (e.g. minimal cost, maximal safety) attributes are transferred to **objectives.**
- **'Ideal' y^*** is a design (usually infeasible) with the coordinates of the best achieved quality for each objective.

Concept of non-dominance (Figs. 1a, 1b):

- The subspace $\mathbf{Y}^{\mathbf{N}}$ of **non-dominated or Pareto optimal or efficient designs** can be identified when designer's preference structure is applied to designs (points) in $\mathbf{Y}^{\geq}$.

- **Only those designs (usually only a small fraction of feasible designs) are of interest to designer since they dominate all other feasible designs.**

Concept of non-dominance

- **Preference** is a binary relation stating that design $\mathbf{y}^i$ is preferred to design $\mathbf{y}^j$.

- **"Better set"** can be defined with respect to given design $\mathbf{y}^0$ if all its elements are preferred to $\mathbf{y}^0$.

- Conversely, the **"worse set"** can be formed containing all designs that are worse than $\mathbf{y}^0$ in all attributes i.e. dominated by it.

- For the preference 'more is better' it is easy to visualize the "worse set" to e.g. design $\mathbf{y}^P$ (see Fig. 1b) as a negative cone with an apex in $\mathbf{y}^P$ containing points left of line y_1^P and below the line y_2^P.

- Finally, the set of non-dominated designs $\mathbf{Y}^{N}$ is defined as a set of designs that have no “better set“, hence they are not dominated by any design.

- Alternatively, design is non-dominated if it is better than any other design in $\mathbf{Y}^{\geq}$ in at least one objective.

- Points in $Y^{\mathbf{N}}$ have their design variable description in $\mathbf{X}^{\mathbf{N}}$ (see Fig. 1a).

Inclusion of subjectivity

Inclusion of subjectivity*- see Figs. $1b_1$ and $1b_2$* is basic to realistic decision-making. It implies:

- Subjective comparison of various designs can be performed using **fuzzy functions $U_i(y_i)$.**

- Membership grade (satisfaction level) $\mu_i=U_i(y_i)$ has range 0-1 (see dotted line).

- In **vibration problems** this function may consist of e.g. series of the inverse bell shaped functions centered at excitation frequencies ($\mu_i=0$ for design in resonance). Concept is widely used in DSP.

Inclusion of subjectivity

- Determination of subjective importance of different attributes can be based on **weighting factors** w_i (**P**).
- The **bi-attribute preference matrix P**=[p_{ij}] contains ratios of subjective importance of attribute y_i compared to y_j.
- Combination of subjectivities for attribute i can be achieved as e.g. product

$$u_i(y_i) = w_i(\mathbf{P})\, U_i(y_i) \; .$$

Inclusion of subjectivity

Subjective metric space- *(see Fig. 1c)* is formed by using mappings

$$\mathbf{u} : \mathbf{Y}^{\geq} \rightarrow \mathbf{M}^{\geq}$$

- There now exists possibility of the **introduction of metric** (distance measure) since all attribute values

$$m_i = u_i(y_i) \qquad \mathbf{m} = \{m_i\}$$

are **normalized and scaled to their relative importance**.

- Subjective criteria functions **u** enable natural 'more is better' preference structure and make it even easier to filter the subset of non-dominated designs $\mathbf{M}^N$ from $\mathbf{M}^{\geq}$ and then generate corresponding $\mathbf{Y}^N$ and $\mathbf{X}^N$.

Value functions

Value functions **v** (utility fn. see Yu, 1985) are defined as mappings $l_i = v_i(\mathbf{m})$.

- Vector $\mathbf{l}^k = \{l_i\}$ contains values obtained from different value functions including in its formulation the subjectivity of designer and others involved in DM.

- The **iso-value contours** l_i = const. can be visualized in **M**-space.

- These contours (like in geography), may exhibit multiple peaks. Some of those peaks correspond to local minima/maxima and some are global i.e. best for the entire $\mathbf{M}^{\geq}$.

- Note that optimum in constrained problems is often achieved on **boundaries of feasible region.**

Distance norms (metrics)

Distance norms (metrics) $\mathbf{L_p}$ are commonly used as value functions.

- Distance to the specified target design m^*(e.g. ideal design) is given by standard expression

$$L_p(\mathbf{m}^k) = [\sum|m_i^k - m_i^*|^p]^{1/p}\,.$$

- Exponent p in the norm definition is taken as

1	Metropolitan n.
2	Euclidean n.
∞	Chebisev n.

Distance norms (metrics)

- The **iso-value contours** for given distance norms are (see Fig.1c):

  (1) straight lines $\sum m_i$=const for L_1,

  (2) circles around $\mathbf{m}^*$ for L_2

  (3) m_i=const for L_∞ .

- The non-dominated design for min L_∞ (marked ■) can be linked to so-called ‘fuzzy optimum’ i.e. a design for which the minimal m_i in $\mathbf{m}^k$ is maximal.

Selection

Selection of preferred design-see Fig. 1d :

- Final selection of **preferred design** P: $\mathbf{d}^P = \{\mathbf{x}^P, \mathbf{y}^P, m^P, \mathbf{l}^P\}$ requires the calculation of values $\mathbf{l}^k$ for all designs in $\mathbf{M}^N$.

- Since final decision is made only on the basis of $\mathbf{l}^k$, all l_i can be put on same axis for each design k.

- A **set of parallel axis** for all candidate designs can be displayed to facilitate final subjective decision.

- Parallel axes can also correspond to all l_i. Lines connecting specific designs on all axes are used to facilitate ranking of designs.

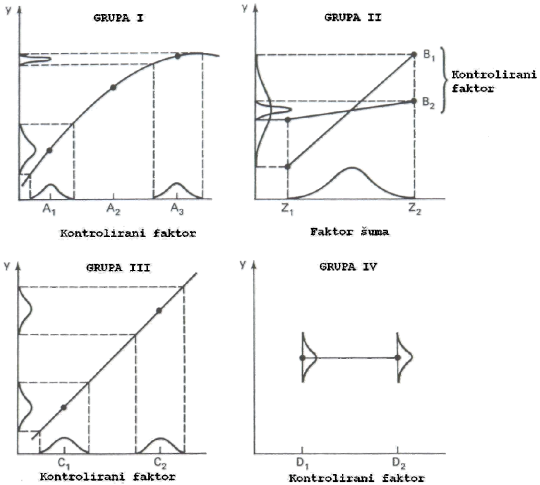
Sensitivity analysis, Robustness and Stochastic Characteristics:

- For technical systems the **existence of solution** is often guaranteed but not its **uniqueness and stability.**

- Many parameters, held constant during optimization process, are **subject to uncertainties** causing variations of the values in the criteria set **Y** and/or violation of constraints (unfeasible designs).

Robustness

Robustness is defined as **insensitivity (or stability)** with respect to such changes.



Design mapping-see Figs.1a,1d :

- Composite value function

$$l_i = r_i(\mathbf{x}^k),$$

built from subjective criteria functions

$$r_i(\mathbf{x}^k) \equiv v_i\,(\mathbf{u}(\mathbf{a}(\mathbf{x}^k))),$$

maps the feasible designs subspace $\mathbf{X}^{\geq}$ into the designer's selection set $\mathbf{L}$.

- The obtained values $\mathbf{l}^{\mathbf{k}}$ (cost, weight, etc.) are used for the final evaluation of the design.

Design mapping

- The described mapping $\mathbf{r} : \mathbf{X}^{\geq} \rightarrow \mathbf{L}$ is called ***evaluation mapping.***

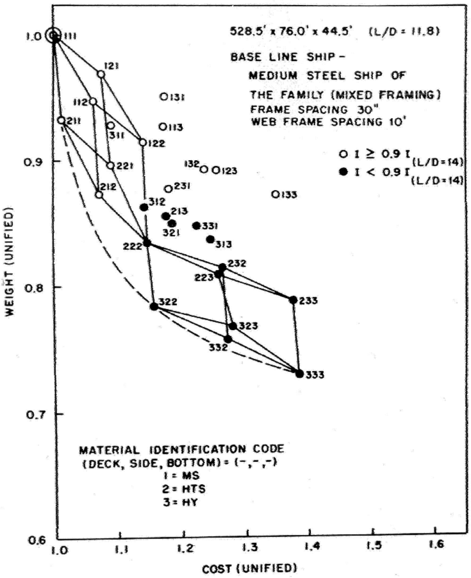
But in the design process the designer's task is to determine values of design variables $\mathbf{x}^{\mathbf{A}}$ for given aspired values $\mathbf{l}^{\mathbf{A}}$ of cost, weight, etc.

- Therefore the construction of the inverse or ***design mapping***

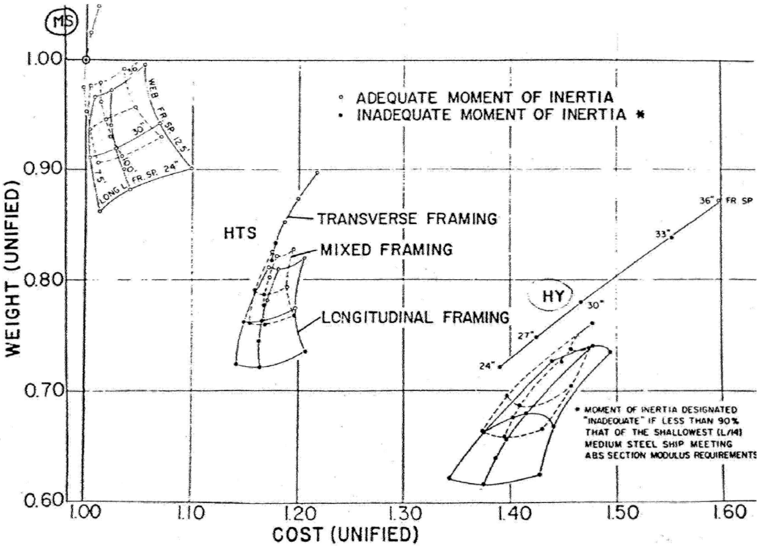
$$\mathbf{r}^{-1} : \mathbf{l}^{\mathbf{A}} \rightarrow \mathbf{X}^{\geq},$$

that maps designer's aspirations to design parameters, **is the essence of design**.

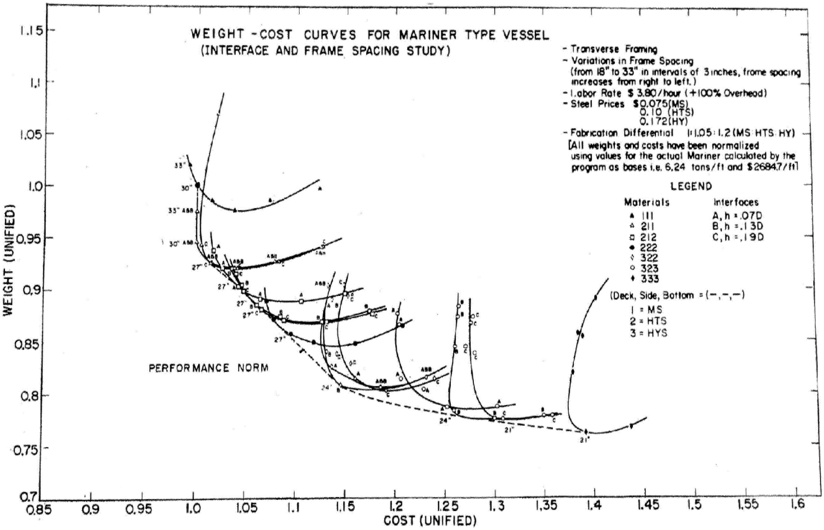
Examples of Attribute space (Evans 1974)



Examples of Attribute space (Evans 1974)



Examples of Attribute space (Evans 1974)



Examples of design problem definition regarding 4 ship types from Lecture 1a.

MODELS	VARIABLES	PARAMETERS	ATTRIBUTES	CONSTRAINTS
1 CAR CARRIER	SCANTLINGS	TRANSV. STRUC. (CLASSIC, HINGE)	COST	C.S. RULES
	MATERIAL		SAFETY	ULT. STRENGTH
	STIFFENING TYPE	PILLAR SYST.	DWT	CLEARANCES
2 LIVESTOCK CARRIER	SCANTLINGS	BRIDGE STRUCT	COST	C.S. RULES
	MATERIAL	STERN STRUCT	SAFETY	ULT. STRENGTH
	STIFFENING TYPE	PILLAR SYST.	MAINTENANCE	RACKING
3 CRUISE SHIP	SCANTLINGS	RECES POSITION	COST	C.S. RULES
	MATERIAL	TBKHD, LBKHD	SAFETY	ULT. STRENGTH
	STIFFENING TYPE	SIDE OPENINGS	VCG	WINDOWS
4 TANK-CAR CARRIER	SCANTLINGS	DECKHOUSE LGT	COST	C.S. RULES
	MATERIAL	DH. MATERIAL	SAFETY	ULT. STRENGTH
	STIFFENING TYPE	DH. LAYOUT	ROBUSTNESS	CLEARANCES

Lecture 1c

MAESTRO PRESENTATION