LECTURE 3c

Emergence of new generation of structural optimization techniques

Basic trends are:

→ Multiple Criteria Decision Making (MCDM) is accepted as the only realistic general approach. Selection of MADM (Multi Attribute) or MODM (Multi Objective) formulation depends on dimensionality and mathematical complexity of the problem (see bellow);

→ Computer speed is used for generation of large number of design variants. Parallel processing is also an important option for such 'workload';

Basic trends

→ Approximate problem formulation is based on sensitivity and perturbation analysis.

→ Emergence of second generation of approximation techniques implies usage of intermediate variables,

→ Move limits in such processes can be raised reducing drastically number of FEM reanalysis (main measure of process efficiency);

Basic trends

→ Metamodeling of criteria functions or entire subspaces (e.g. X^N) is inexpensive-to-run approximation of expensive-to-run computer analysis. It is obtained using: (1) response surfaces (2) neural networks,

→ Neural networks are special form of response surfaces using nested squashing function. Kriging technique is a combination of fixed criteria function and departure from it described as realization of stochastic process with zero mean and spatial correlation function.

Basic trends

- → Synergetic Multi Disciplinary Optimization (MDO) or Multi Discipline Feasible (MDF) combines parameters and criteria from hydrodynamics, structures, production etc.;
- → Multilevel problem decomposition (hierarchical or not) to global and local (subsystem) levels is a must for large MDO design problems. Basic to such developments are the 'agentification' of DSP to enable communication (knowledge exchange), and decomposition strategy where sensitivity analysis can play dominant role.
- → Emerging techniques for large scale problems are: Simultaneous Analysis and Design (SAND), Nested Analysis and Design (NAND), All-at-once (AAO), Individual Discipline Feasible (IDF) and multilevel Concurrent Subspace Optimization (CSSO).

→ The linear combinations of prescribed (basis) designs can drastically reduce number of design variables, problem complexity and even enable coupling of shape and scantling optimizations;

→ Efficient FEM re-meshing (using sensitivity information) gives the quality response information and cuts down FEM time.

Manipulations and Solution strategies - MODM approach

The techniques for the highly non-linear and high-dimensional problems are necessarily leading to variety of methods in operations research closely tailored to the characteristics of objective and constraint functions of the problem at hand.

Design mapping in MODM is usually transformed to the standard mathematical programming formulation:

$$\max \mathbf{r}(\mathbf{x}) \text{ such that } \mathbf{x} \in \mathbf{X}^{\geq}.$$

If a value function combining multiple objectives can be constructed the methods for single compound objective could be used e.g. compromise and goal programming methods

MODM formulations

They can be further manipulated as follows:

(M1) Problem is projected (partitioned) to the subset of design variables (others fixed).

(M2) Linearizations and metamodeling techniques can replace failure surfaces or their envelopes.

(M3) Dualization of the problem combines objective function and constraint functions via Lagrange multipliers. They are the dual variables entering the problem linearly. Many practical and successful formulations are given in dual form:

Basic MODM strategies

Strategies used to solve manipulated problems are:

(S1) Iterative and piecewise strategy (leading to sequence of simple problems e.g. feasible directions, penalty function approach, etc.),

(S2) Relaxation strategy (temporarily removing some constraints e.g. in dualization),

(S3) Restriction s. (fixing of some variables temporarily to zero e.g. in linear programming).

Standard useful combinations

{Manipulation(s)/Strategies} are:

- {projection, outer linearization/ relaxation e.g. cutting plane},
- {projection/piecewise},
- {inner linearization/restriction e.g. Dantzig-Wolfe},
- {projection/feasible directions} or {dualization/feasible directions}.

Methods described under (M3) are basically application of {dualization, linearization / relaxation}.

Hybrid model (MODM + MADM)

MODM

- dual formulation of sequential linear programming
- accumulation of linearised constraints for nonlinear feasibility criteria
- special linearisation technique (including 2^{nd} order terms) in generating failure hyperplanes.
- application of (dualisation, linearisation/relaxation).

MADM

- global optimizer is also based on (dualisation, linearisation /relaxation) paradigm but with constraint set obtained through MADM generated response surface.

MADM approach

The selection of the best design is done **among the discrete number of design alternatives via straightforward evaluation**.

The increased speed of workstations is used to model the complex design problem as a **multiple evaluation process** by intentionally creating a large number of design variants through random search methods as the simplest and most robust of non-gradient techniques.

If sufficient density of non-dominated points is generated a **'discrete' inversion** of the evaluation mapping is obtained for the most important parts of design space.

Therefore, it is possible to replace optimization oriented MODM approach with much simpler MADM.

MADM approach

It implies:

- **Generation, evaluation and filtering of non-dominated designs**
- **Selection procedure in metric space**

In this way, problems of discrete variables and multiply connected domain, prohibiting application of MODM methods, become irrelevant.

MADM approach is particularly efficient for the considered problem of concept design and design of subsystems in preliminary/initial design.

Design generation and evaluation strategies

MADM method is used for the generation of good parent designs on the non-dominated hyper-surfaces X^N, Y^N or M^N.

The emerging computational paradigm is to follow processes in nature ("superb designer") and methods are modeled accordingly: Genetic Algorithms, Evolution Strategies e.g., etc.

They are non-dominance driven sequential and adaptive. Local and global search methods may differ and hybrid methods may emerge in the future).

Six approaches of stochastic search are presented, three of which are non-dominance driven, sequential and adaptive.

Design generation and evaluation strategies.

The emerging computational paradigm is to follow processes in nature (“superb designer”) and the last three methods are modeled accordingly. These methods are also more robust to local minima.

Strategies S1 - S3 are used in DeMak optimization module and can be interactively selected.

On global level the predictive task is performed by metamodeling techniques.

Design generation and evaluation strategies

(S1) Crude Monte Carlo sampling by the random number generator (RNG) in design space to get n non-dominated designs in t trials. It is used as first step in S2 and S3 and multiple start in MODM.

(S2) Sequential adaptive random generation of non-dominated designs.

- Designs surviving feasibility tests are tested for dominance in the Pareto sense.
- They are used as centers of subspaces (minicubes) in design space for further sequential ("chain") generation of non-dominated candidates for final design selection e.g.

- Basic difference to S1 is adaptive bounds as functions of current non-dominated point x^k.

Design generation and evaluation strategies

(S3) Fractional Factorial Design (FFD), using orthogonal arrays (OA) constructed from the Latin squares are applied for efficient generation of designs.

- They have proven efficient in higher cycles of adaptive design generation in subspaces around the non-dominated designs.
- The number of factor (variable, parameter) levels is from 2 to 5. Orthogonal arrays (L9, L27) with 3 levels accommodating up to 13 design variables are used.
- They permit parallel efficient building of response surfaces used for global problem.

Design generation and evaluation strategies

(S4) Genetic Algorithms (GA) include

(a) crossover i.e. exchange of parts of chromosome contents (string of decimal or binary values of design variables **x**),

(b) mutation of chromosome content and

(c) statistical selection of surviving designs.

- GA are modeled following natural selection with Darwinian survival of the fittest. They correspond to randomized adaptive search. They differ from S1-S3 by coding of the parameter set, not the parameters themselves. They use probabilistic and not deterministic transition rules regarding design fitness.
- Immune Network Modeling with the antigen strings and generalist antibody strings can be used to coordinate subsystems into cooperative system. Fitness function is bit-by-bit match-score obtained from comparison of strings.

Design generation and evaluation strategies

(S5) Evolution Strategies (ES) (crossover not very important) e.g. are similar to S4. Strategies 4-5, like S2, search from a population of points (one generation is recombined to generate new one), not a single point.

Different heuristic methods (Tabu search, Expert and Classifier systems) can be used for streamlining and guiding the design process using developed population of designs to develop new rules and/or actions.

Design generation and evaluation strategies

(S6) Simulated annealing is patterned on the physical process of optimum layout of molecules due to annealing

- The objective function of the optimization problem is taken as the energy corresponding to a given system state (i.e. design).
- The design variations ($\mathbf{x}^k$) for given 'temperature' (process control parameter) are treated as the probable states with Boltzman distribution.
- Number of random variations at each temperature and the rate at which temperature is lowered is called annealing schedule.

Design generation and evaluation strategies

- The strategies S1-S6 are used combined with predictive task performed by meta-modeling techniques.

- Parallel processing is easily applicable to S1-S6 strategies with the ‘processor farming’ (independent work) applied in generation of feasible designs. The algorithmic parallelism of processors can speed up the process of filtration of non-dominated designs or the GA population selection.

- Since S4-S6 are basically unconstrained optimization techniques the constraint set **g(x)** has to be included e.g. penalty function approach:

$$f^P(\mathbf{x}) = f(\mathbf{x}) + c\,P(\mathbf{g}(\mathbf{x})),$$

  Immune simulation (gene repair) is also used in S4-5 to generate feasible 'children'.

Selection strategies in MADM

The selection of the best design is done among the discrete number of design alternatives via straightforward evaluation.

Simple and fast calculation of $\mathbf{L}^{N}$ is performed for known $\mathbf{M}^{N}$. Minimization problem is thus reduced to simple comparison of $\mathbf{l}^{k}$ values.

L4A Safety and Stochastic Characteristics

Attributes should provide measure of quality that is enabling relative comparisons of the design variants implying that only the order of variants have to be preserved. Following safety measures (design attributes) were investigated:

→ Subsystem reliability attribute (SSR) defined as upper Dietlevsen bound of serviceability criteria to account for the probability of failure connected to repair costs (Zanic et al. 1993).

→ System reliability attribute (SCR) is obtained via multilevel β-unzipping procedure (Murotsu 1986) for the racking collapse of the cross-section.

→ Deterministic maximal racking collapse load attribute (DCL) can be used as measure of safety and structural redundancy based upon sequences of local failures before the system collapse (singularity of downgraded stiffness matrix).

→ Robustness attribute (TSN) is defined as insensitivity (or stability) with respect to the environmental (parameter) changes. It is a new design attribute that should be maximized in the multi-attribute design procedure.

A metric developed by Taguchi (Montgomery 1991) is the ratio of the mean of the attribute value μ (resulting from the values of design variables and parameters) to the variation (resulting from uncertain parameter values) measured via standard deviation σ. It is the ratio of predictability versus unpredictability. For the 'nominal is the best' attribute, the Taguchi's signal-to-noise ratio reads:

$$SN = 10 \log(\mu^2/\sigma^2) = 10 (\log \mu^2 - \log \sigma^2) \;;$$

However, an incoherent answer can also be obtained since the ratio is a function of the attribute's mean and its variance, i.e. sometimes conflicting measures of position and dispersion.

In the presented model, the calculation, applied for each substructure, is summarized in the sequel:

1. Choose parameter (σ_x, σ_y, τ, p) levels broad enough to include realistic range of possibilities;

2. Define variable levels for the current cycle;

3. Perform N global experiments (designs) using orthogonal arrays to generate designs $\mathbf{x}^{\mathbf{i}}$, $i = 1, ..., N$;

4. Perform local experiments j = 1, ..., L for each global experiment i to calculate measure of robustness for affected attribute aij = {ĝi[xi,(σx, σy, τ, p)j]}, taking into account small size of sample when calculating estimates of variance (σ2) and approximate mean square (μe2) of attribute for the global experiment i:

$$M = \sum aij/N \; ; \; \sigma 2 = (\sum aij2 - N \cdot M2)/(N-1) \; ;$$

$$\mu_e{}^2 = (N \cdot M^2 - \sigma^2)/N \; ; \; (S/N)_i \equiv a_4(\mathbf{x}^i) = 10 \log (\mu_e{}^2/\sigma^2) \; ;$$

5. Calculate level variations of attribute for each level of design parameter to estimate influence of parameter on the attribute robustness;

6. Calculate compound measure of the design robustness for normalized attributes (via membership grade) practical for the design generation phase.

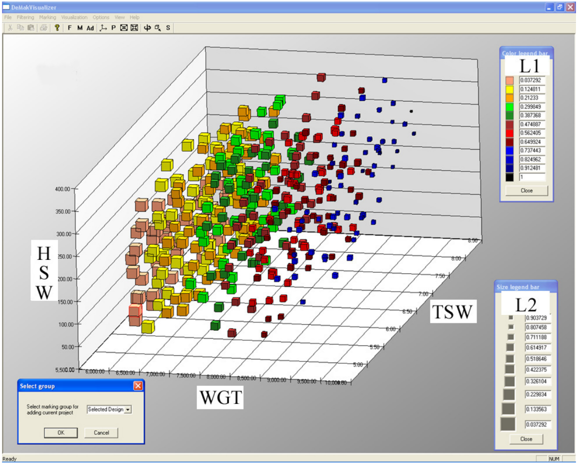
7. Eliminate dominated designs with respect to all design attributes including robustness attribute;

8. Select automatically the preferred design of substructure based on the utility function (u) or value function (v = e.g. distance norms L_1, L_2, L_∞);

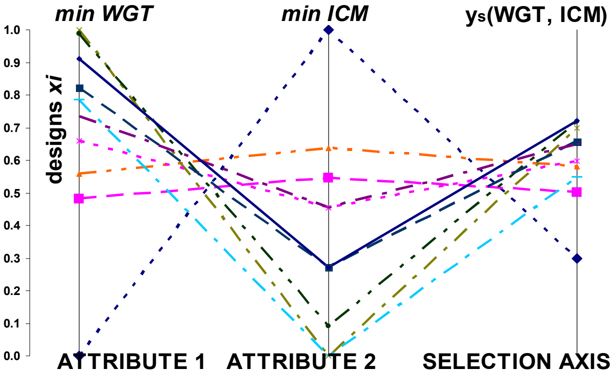
9. Present all design variants or design attributes in 5D design space or on parallel axes for the final interactive design selection.

Visualization

is the most powerful tool for designer's understanding of the DS problem. Stratified distances from the ideal design (Fig. 15c), calculated by L_p metric can be used as a means of visualizing multidimensional space of design attributes and/or free variables It generates expert knowledge about the problem for all participants involved, helps the designer to identify advantageous combinations of variables, other feasible options and clusters of non-dominated designs thus enabling realistic decision support to the principal and structural designer



5D view into substructure design/attribute space.



View of attributes on parallel axes for substructures.