LECTURE 4a

Subjectivity

(1) Intra-Attribute Preferences and Normalization

- Intra-attribute preference reflects the relative importance of the different values of the same attribute.
- As compared to crisp requirements (or constraints), fuzzy approach softens the sharp transition from acceptable to unacceptable.
- At the same time the values of design attributes should be normalized in order to make them commensurable in multidimensional space.

Attribute Normalization

Normalization of attribute *j* values (e.g. vector or linear scale)

$$y_{ij} = \frac{Y(i,j)}{\sqrt{\sum_{k=1,NA} Y^2(k,j)}}$$

or

$$\mathrm{y_{ij}} = \frac{\mathrm{Y(i,j) - Y(i,*)_{min}}}{\mathrm{Y(i,*)_{max} - Y(i,*)_{min}}}$$ for maximization (* : j=1,..ND)

$$\mathrm{y_{ij}} = \frac{\mathrm{Y(i,*)_{max} - Y(i,j)}}{\mathrm{Y(i,*)_{max} - Y(i,*)_{min}}}$$ for minimization

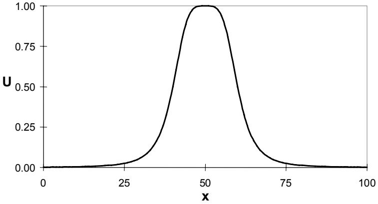
where:

Y = [Y(i,j)] is a decision matrix of attributes of non-dominated designs (i = 1,.,NAtributtes; j = 1,.,NDesigns)

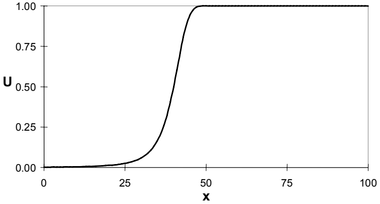
y = [y_{ij}] normalized decision matrix

Membership grade: preferences/attribute normalization

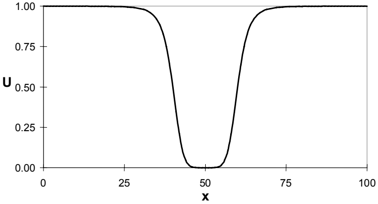
Ω-type - Attracting



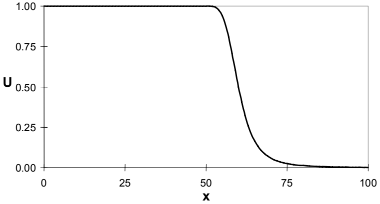
S-type - Ascending



U-type - Averting



Z-type - Descending

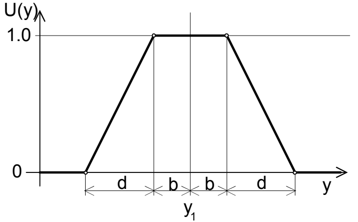


Four principal types of membership grade functions

Membership grade functions (Grubisic et al. (1997)

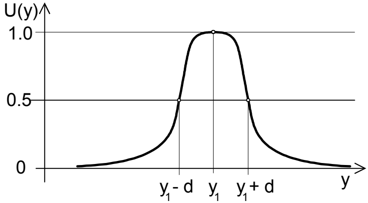
Linear type

$$U(y)=\begin{cases}0 & \Rightarrow\ y\le y_1-b-d\\ 1+\dfrac{y-(y_1-b)}{d} & \Rightarrow\ y_1-d-b<y<y_1-b\\ 1 & \Rightarrow\ y_1-b\le y\le y_1+b\\ 1-\dfrac{y-(y_1+b)}{d} & \Rightarrow\ y_1+b<y<y_1+b+d\\ 0 & \Rightarrow\ y_1+b+d\le y\end{cases}$$



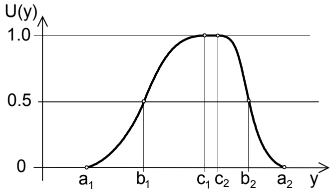
Generalized Nehrling type

$$U(y)=\frac{1}{1+\left|\dfrac{y_1-y}{d}\right|^{N}}$$



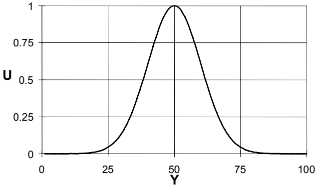
Novak type

$$U(y)=\begin{cases}\dfrac{(y-a_1)^2}{(b_1-a_1)\cdot(c_1-a_1)} & \Rightarrow\ a_1\le y<b_1\\[2ex] 1-\dfrac{(y-c_1)^2}{(c_1-b_1)\cdot(c_1-a_1)} & \Rightarrow\ b_1\le y<c_1\\[2ex] 1-\dfrac{(y-c_2)^2}{(b_2-c_2)\cdot(a_2-c_2)} & \Rightarrow\ c_2<y\le b_2\\[2ex] \dfrac{(y-a_2)^2}{(a_2-b_2)\cdot(a_2-c_2)} & \Rightarrow\ b_2<y\le a_2\end{cases};\begin{cases}1\Rightarrow c_1\le y\le c_2\\ 0\Rightarrow a_1\ge y\ge a_2\end{cases}$$



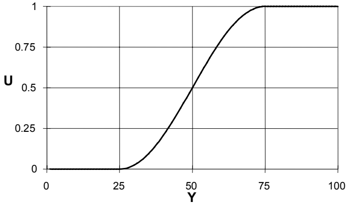
Gauss type

$$U(y)=\exp\left[-0.5\cdot\left(\frac{y-y_1}{\sigma}\right)^2\right]$$



Sinus type, ascending

$$U(y)=\begin{cases}0 & \Rightarrow\ y\le y_{1/2}-d\\ 0.5\cdot\left[1+\sin\left(\pi\cdot\dfrac{y-y_{1/2}}{2\cdot d}\right)\right] & \Rightarrow\ y_1-d<y\le y_{1/2}+d\\ 1 & \Rightarrow\ y>y_{1/2}+d\end{cases}$$



Inter-Attribute Preferences

- Design attribute values serve as a basis for selection of the final design among all non-dominated designs.
- All attributes are not equally influential or important.
- It is necessary to apply weighting to reflect their relative importance.
- One solution is to obtain weighted membership grade by multiplication with a weighting factor reflecting designer's preference.
- In this respect Saaty's method, based on preference between attributes a_i and a_j was found suitable. The rating preference code from 1 to 9 is used.

Inter-Attribute Preferences

- The consistent relative significance of attributes is obtained as an eigenvector of subjective preference matrix. The matrix is formed as a result of pair wise comparisons of attributes.

- Preference matrix is defined as:

$$\mathbf{P} = [p_{ij}] \text{ where } p_{ij} = p_i / p_j \,; \quad p_{ij} = p_{ji}^{-1}$$

Preference code p_{ij} (row i, column j) for attribute pair a_i, a_j

p_{ij} (j>i) (a_i preferred)	PAIRWISE PREFERENCE	p_{ij} (a_j preferred)
1	Equally important	1
3	Slightly preferred	1/3
5	Strongly preferred	1/5
7	Demonstrably preferred	1/7
9	Absolutely preferred	1/9

Inter-Attribute Preferences

- Importance vector is defined as

$$\mathbf{p} = \{p_i\} \;;$$

where: p_i importance of attribute i;
p_j importance of attribute j;
$i, j = 1, \ldots,$ NA (number of attributes).

- If the designer is fully consistent he/she will obtain:

$$\mathbf{P} \cdot \mathbf{p} = NA \cdot \mathbf{p}$$

- In the case of the inconsistency of judgment the normalized eigenvector $\mathbf{\Lambda} = \{w_i\}$, corresponding to the eigenvector $\mathbf{p}$ of the largest eigenvalue (λ_{max}) of the problem:

$$(\mathbf{P} - \lambda\ \mathbf{I})\ \mathbf{p} = 0 \;\; ; \;\; \mathbf{I} = \text{unit matrix} \;;$$

is used as a vector of relative weights of attributes.

Inter-Attribute Preferences

- Consistency of the preference may be estimated by the criterion:

$$C = (\lambda_{max} - NA) / (NA - 1) < 0.1.$$

- Alternatively, the normalized geometric mean of the row of the preference matrix may be used as weight :

$$w_i = (\Pi_{j=1,..NA}\, p_{i\,j})^{1/NA}\ ;$$

Value and utility functions v( )

- They are defined as mappings $l_i = v_i(\mathbf{m})$.
- Vector $\mathbf{l}^k=\{l_i\}$ contains values obtained from different value functions and includes in its formulation the subjectivity of designer and others involved in decision making.
- The iso-value contours l_i = const. can be visualized in **M**-space. These contours (like in geography), may exhibit multiple peaks. Some of those peaks correspond to the local minima/maxima and some are global i.e. the best for the entire $\mathbf{M}^{\geq}$.
- Note that optimum in constrained problems is often achieved on boundaries of the feasible region.

Distance norms L_p

- Distance norms (metrics) L_p are commonly used as value functions.
- Distance to the specified target design m^*(e.g. ideal design) is given by standard expression:

$$l_i \equiv L_p(m^k) = [\sum|m_i^k - m_i^*|^p]^{1/p} \; ;$$

- Exponent p in the norm definition is taken as 1, 2 (Euclidean n.) or ∞ (Chebisev n.).
- The iso-value contours for given distance norms are:

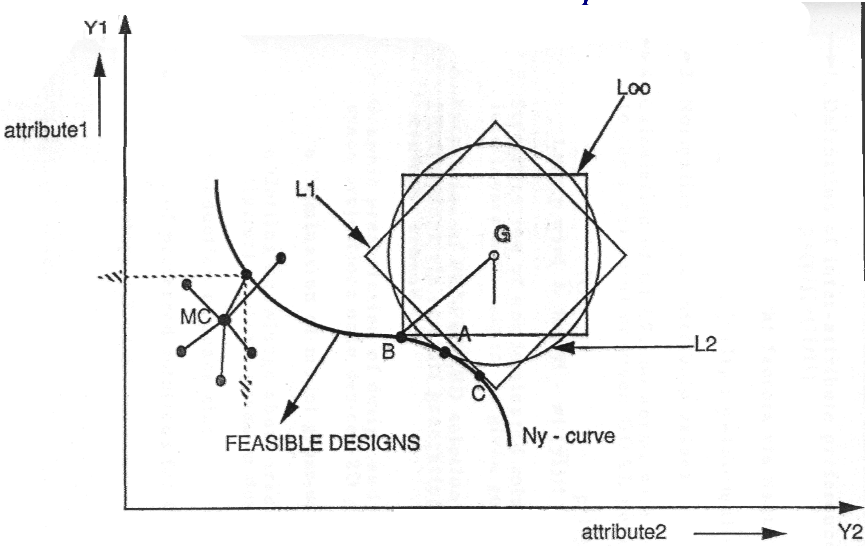
(1) straight lines $\sum m_i$=const for L_1,

(2) circles around $\mathbf{m}^*$ for L_2 and

(3) m_i=const for L_∞ .

The non-dominated design for min L_∞ (marked ■) can be linked to so-called ‘fuzzy optimum’ i.e. a design for which the minimal m_i in $\mathbf{m}^k$ is maximal.

Distance norms L_p



Isovalue curves for different metrics

Principal Steps During Selection Based On Subjectivity

- Interactive definition of the preference information regarding relationship between attributes (or alternatively preference information on design variants). Selection among ND designs is then very fast.
- Extraction of weight factors from subjective preference matrix via Saaty's method (Saaty, 1977). Least squares method or entropy method can be alternatively used. The main problem is to achieve consistency of preference definition.
- Selection of attribute value type (direct, membership grade formulation).
- Normalization of attribute values (vectorial or linear scale):
- Calculation of L_1, L_2 (Euclid) and L_∞ (Chebyshev) norms with respect to given ideal point (mI) or prescribed goal for all non-dominated designs (m), with given attribute weights (w).

Principal Steps

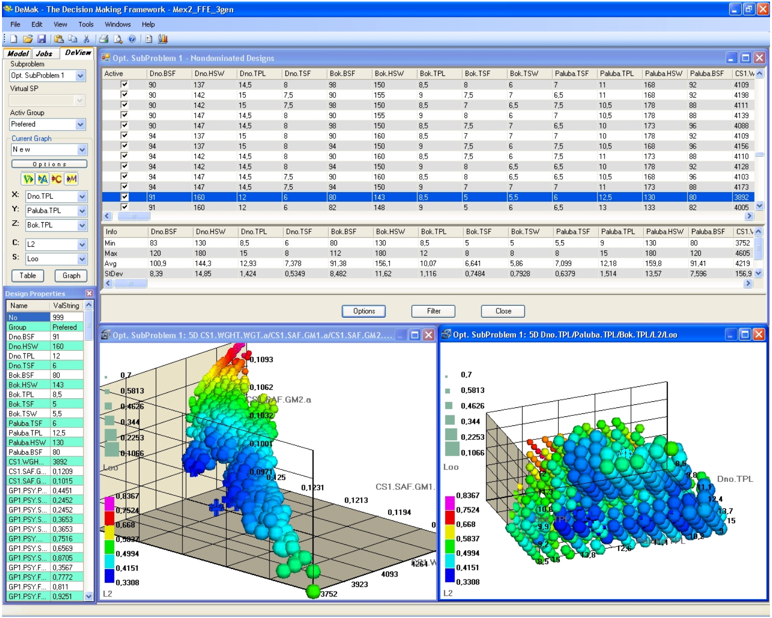
- Construction of other value functions, if required.
- Stratification of the set of non-dominated solutions into layers according to the value function (e.g. L_1, L_2 or L_∞ norms or other). Stratified X or Y space can be used for:
- Graphic presentation via stratified X or Y space;
  - experiments in interpolation (e.g. design variable as function of design attribute values in specified limited stratum).
- Extraction of preferred solutions according to given preference structure. In this way designs of minimal distance from ideal or the other prescribed goal are obtained and displayed.

Visualization and Selection of Prefered Designs

- *Visualization* is the most powerful tool for designer's understanding of the DSP.

- Stratified distances from the ideal design, calculated by L_p metric can be used as a means of visualizing multidimensional space of design attributes and/or free variables.

- It generates expert knowledge about the problem for all participants involved, helps the designer to identify advantageous combinations of variables, other feasible options and clusters of non-dominated designs thus enabling realistic decision support to the principal and structural designer.


DeMak - The Decision Making Framework - Mex2_FFE_3gen
File Edit View Tools Windows Help
Model Jobs DeView
Subproblem
Opt. SubProblem 1
Virtual SP
Activ Group
Prefered
Current Graph
New
Options
Table
Graph
Design Properties
Opt. SubProblem 1 - Nondominated Designs
Active Dno.BSF Dno.HSW Dno.TPL Dno.TSF Bok.BSF Bok.HSW Bok.TPL Bok.TSF Bok.TSW Paluba.TSF Paluba.TPL Paluba.HSW Paluba.BSF
Info Min Max Avg StDev
Options Filter Close
Opt. SubProblem 1: 5D CS1.WGHT.WGT.a/CS1.SAF.GM1.a/CS1.SAF.GM2....
Opt. SubProblem 1: 5D Dno.TPL/Paluba.TPL/Bok.TPL/L2/Loo
Dno.TPL
Loo
L2

Graphic representation of design and attribute space and interaction with designer

- The process of design selection is basically interactive since designer would change and refine his preferences (sensitivity study).

- Different methods of selection of preferred solution will be tried concurrently. It is of great importance to give to the designer some insight into multidimensional design or attribute space to help in guiding his decisions and graphic processing is added to the design process.

- All non-dominated points can be viewed in diagrams specified by any combination of variables/attributes. The step size for each axis is specified by the designer.

Graphic representation

- There are 4 types of diagrams with the following combinations:

(1)	attribute - attribute	Y space projection
(2)	variable - attribute	cross projection
(3)	variable - variable	X space projection
(4)	multiple diagram	combination of (1), (2), (3)

The first group relating various attributes may be used as a guide to isolate advantageous regions and to gain impression of what may be the penalty for departing from the optimum.

Graphic representation

The second group is used to analyze the influence of any variable upon any attribute.

- The likely effect the change of the variable will have on the attribute value may be quantitatively predicted.

- It is particularly useful to identify range of the variable for favorable attribute values and so be able to reduce range within which variables are to be generated in the next try.

- In this way the density of designs in the vicinity of optimum may be increased. Sometimes it will be demonstrated that there are 'holes' in a region of the design space, or narrow 'strips' or 'corridors'.

- This behaviour demonstrates influence of all constraints and characteristics of design model response.

Graphic representation

The third group is common to all designers, relating principal dimensions and coefficients.

- Upon inspecting diagrams the limits of all variables are reviewed and their span modified so as to concentrate further procedure in the proximity of best designs only.
- Alternatively, a new batch of non-dominated designs may be generated if more points are needed for decision making in the zone of particular interest.

The following features were found useful in selection process:

- Non-dominated solutions (or all generated feasible solutions if saved) can be examined.
- Stratification of design points is marked via signs diminishing in size for given number of strata.

Graphic representation

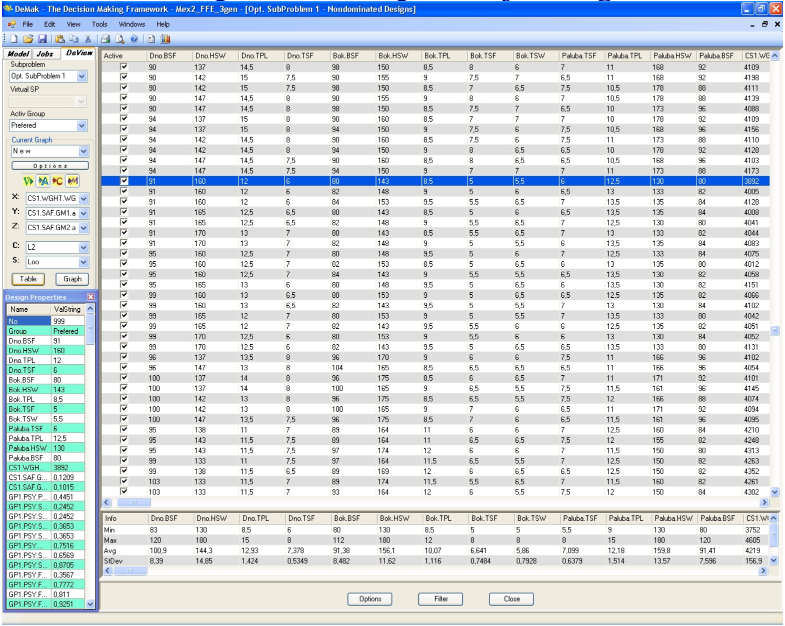
- Characteristic points (extremes, preferred points, user specified) are marked. All of them are visible in all diagrams. They are used for orientation in subspace of non-dominated solutions.
- Bounds can be applied to cut off from the diagram the points that do not satisfy prescribed values of attributes and variables.
- Selected areas of design and/or attribute space can be zoomed for fine resolution.
- A batch of diagrams can be displayed simultaneously to facilitate comprehension of multi-dimensional spaces, particularly regarding relative position of specified design points.
- Histograms and marginal or joint distributions for specified variables/attributes can be used.

Graphic representation

The graphic capabilities are used primarily for the following purposes:

- Elimination of model gross-errors by using designers' synthetic judgment particularly by inspecting positions of maxima/minima of different attributes.
- Acquiring 'heuristic' knowledge about design and attribute space, particularly trends, spread, clusters of 'good' points, multiple peaks, etc.
- Reducing the size of design space on the basis of insight into location of important points.
- Final step of design process is the selection among selected preferred solutions that can be done by some techniques of ranking for problem at hand.

Complete Description of Design



Name	ValString
No	999
Group	Prefered
Dno.BSF	91
Dno.HSW	160
Dno.TPL	12
Dno.TSF	6
Bok.BSF	80
Bok.HSW	143
Bok.TPL	8,5
Bok.TSF	5
Bok.TSW	5,5
Paluba.TSF	6
Paluba.TPL	12,5
Paluba.HSW	130
Paluba.BSF	80
CS1.WGH...	3892
CS1.SAF.G...	0,1209
CS1.SAF.G...	0,1015
GP1.PSY.P...	0,4451
GP1.PSY.S...	0,2452
GP1.PSY.S...	0,2452
GP1.PSY.S...	0,3653
GP1.PSY.S...	0,3653
GP1.PSY....	0,7516
GP1.PSY.S...	0,6569
GP1.PSY.S...	0,8705
GP1.PSY.F...	0,3567
GP1.PSY.F...	0,7772
GP1.PSY.F...	0,811
GP1.PSY.F...	0,9251

Active	Dno.BSF	Dno.HSW	Dno.TPL	Dno.TSF	Bok.BSF	Bok.HSW	Bok.TPL	Bok.TSF	Bok.TSW	Paluba.TSF	Paluba.TPL	Paluba.HSW	Paluba.BSF	CS1.WG
☑	90	137	14,5	8	98	150	8,5	8	6	7	11	168	92	4109
☑	90	142	15	7,5	90	155	9	7,5	7	6,5	11	168	92	4198
☑	90	142	15	7,5	98	150	8,5	7	6,5	7,5	10,5	178	88	4111
☑	90	147	14,5	8	90	155	9	8	6	7	10,5	178	88	4139
☑	90	147	14,5	8	98	150	8,5	7,5	7	6,5	10	173	96	4088
☑	94	137	15	8	90	160	8,5	7	7	7	10	178	92	4109
☑	94	137	15	8	94	150	9	7,5	6	7,5	10,5	168	96	4156
☑	94	142	14,5	8	90	160	8,5	7,5	6	7,5	11	173	88	4110
☑	94	142	14,5	8	94	150	9	8	6,5	6,5	10	178	92	4128
☑	94	147	14,5	7,5	90	160	8,5	8	6,5	6,5	10,5	168	96	4103
☑	94	147	14,5	7,5	94	150	9	7	7	7	11	173	88	4173
☑	91	160	12	6	80	143	8,5	5	5,5	6	12,5	130	80	3892
☑	91	160	12	6	82	148	9	5	6	6,5	13	133	82	4005
☑	91	160	12	6	84	153	9,5	5,5	6,5	7	13,5	135	84	4128
☑	91	165	12,5	6,5	80	143	8,5	5	6	6,5	13,5	135	84	4008
☑	91	165	12,5	6,5	82	148	9	5,5	6,5	7	12,5	130	80	4041
☑	91	170	13	7	80	143	8,5	5,5	6,5	7	13	133	82	4044
☑	91	170	13	7	82	148	9	5	5,5	6	13,5	135	84	4083
☑	95	160	12,5	7	80	148	9,5	5	6	7	12,5	133	84	4075
☑	95	160	12,5	7	82	153	8,5	5	6,5	6	13	135	80	4012
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☑	99	160	13	6,5	82	143	9,5	5	5,5	7	13	130	84	4102
☑	99	165	12	7	80	153	9	5	5,5	7	13,5	133	80	4042
☑	99	165	12	7	82	143	9,5	5,5	6	6	12,5	135	82	4051
☑	99	170	12,5	6	80	153	9	5,5	6	6	13	130	84	4052
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☑	96	147	13	8	104	165	8,5	6,5	6,5	6,5	11	166	96	4054
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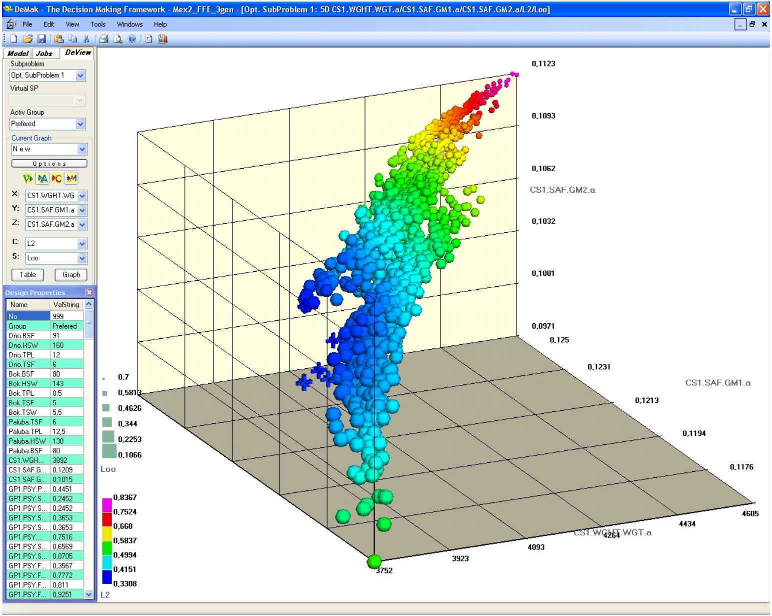
☑	100	137	14	8	100	165	9	6,5	5,5	7,5	11,5	161	96	4145
☑	100	142	13	8	96	175	8,5	6,5	5,5	7,5	12	166	88	4074
☑	100	142	13	8	100	165	9	7	6	6,5	11	171	92	4094
☑	100	147	13,5	7,5	96	175	8,5	7	6	6,5	11,5	161	96	4095
☑	95	138	11	7	89	164	11	6	6	7	12,5	160	84	4210
☑	95	143	11,5	7,5	89	164	11	6,5	6,5	7,5	12	155	82	4248
☑	95	143	11,5	7,5	97	174	12	6	6	7	11,5	150	80	4313
☑	99	133	11	7,5	97	164	11,5	6,5	5,5	7	12,5	150	82	4263
☑	99	138	11,5	6,5	89	169	12	6	6,5	6,5	12,5	150	82	4352
☑	103	133	11,5	7	89	174	11,5	5,5	6,5	7	11,5	160	82	4261
☑	103	133	11,5	7	93	164	12	6	5,5	7,5	12	150	84	4302

Info	Dno.BSF	Dno.HSW	Dno.TPL	Dno.TSF	Bok.BSF	Bok.HSW	Bok.TPL	Bok.TSF	Bok.TSW	Paluba.TSF	Paluba.TPL	Paluba.HSW	Paluba.BSF	CS1.WI
Min	83	130	8,5	6	80	130	8,5	5	5	5,5	9	130	80	3752
Max	120	180	15	8	112	180	12	8	8	8	15	180	120	4605
Avg	100,9	144,3	12,93	7,378	91,38	156,1	10,07	6,641	5,86	7,099	12,18	159,8	91,41	4219
StDev	8,39	14,85	1,424	0,5349	8,482	11,62	1,116	0,7484	0,7928	0,6379	1,514	13,57	7,596	156,9

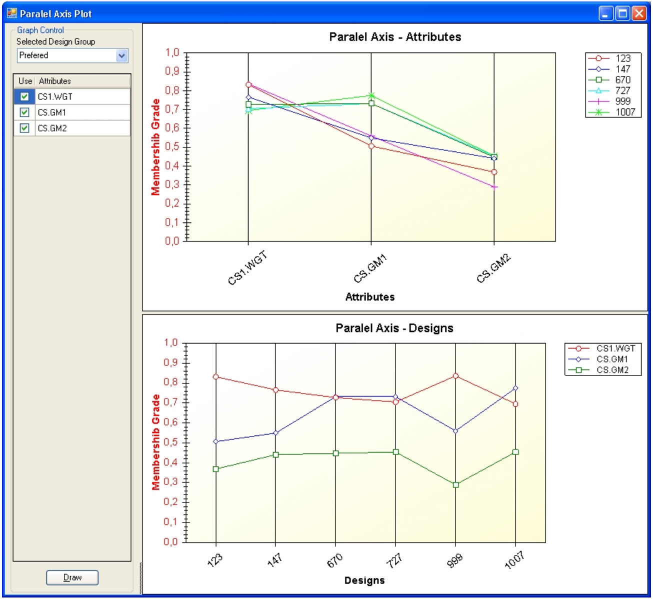
Selection of Preferred Designs

- Attributes of the concept design model should be normalized via membership grade functions and at the same time aspiration levels of the designer are to be formulated.
- Weighting factors should be applied.
- The analysis of subjective weighting factors and membership grade functions is performed for already defined set of non-dominated designs and is therefore very fast and allows for different approaches of decision-makers involved in design of ships.

Visualization in 5D Space



Final Selection in Parallel Axis Plot



Sensitivity Analysis

1. Sensitivity via Lagrange multipliers

$$f_{,x}\,dx + \boldsymbol{\lambda}^{\mathrm{T}} \nabla \mathbf{g} \cdot dx = 0$$

$$df + \boldsymbol{\lambda}^{\mathrm{T}} dg = 0 \quad \rightarrow \quad df = -\boldsymbol{\lambda}^{\mathrm{T}} dg$$

- ❑ Relaxing constraint by an infinitesimal small amount dg_j results in a decrease (i.e. improvement) of the objective.

- ❑ Lagrange multiplier is a measure of how sensitive the optimal solution is w.r.t. modification of the constraint.

❑ A large multiplier means a strong influence of the constraint and vice versa (Note: constraints should be normalized before multipliers are compared).

2. Sensitivity of stiffness in static problems

Fundamental system equation: $\mathbf{K}\,\mathbf{u} = \mathbf{P}^{LC}$

(1) Sensitivity of displacement field for load-case P^{LC} w.r.t. design variable x_i:

$$\left(\mathbf{K}\mathbf{u}\right),_x = \mathbf{P},_x$$

$$\mathbf{K},_x \cdot \mathbf{u} + \mathbf{K} \cdot \mathbf{u},_x = \mathbf{P},_x$$

$$\mathbf{u},_x = \mathbf{K}^{-1}\left(\mathbf{P},_x - \mathbf{K},_x \cdot \mathbf{u}\right)$$

Term in brackets is called pseudo vector (P^{LC*}) for given LC and x_i.

Derivative equation: $\mathbf{K}\,\mathbf{u}_{,xi} = \mathbf{P}^{LC*}$

(2) Sensitivity of constraint g_j w.r.t. design variable x_i

$$g_j,_{x_i} = g_j,_x + g_j,_{\mathrm{u}} \cdot \mathbf{u},_{x_i}$$

$$= g,_{x_i} + g,_{\mathbf{u}} \cdot \mathbf{K}^{-1}\left(\mathbf{P},_{x_i} - \mathbf{K},_{x_i} \cdot \mathbf{u}\right)$$

(3) For element e stresses and their derivatives w.r.t. x are:

$$\sigma = \mathbf{EBu}$$

$$\sigma,_x = \underbrace{\mathbf{EB},_x \mathbf{u}}_{\sigma,_x} + \underbrace{\mathbf{EB}}_{\sigma,_{\mathbf{u}}} \mathbf{u},_x$$

Stiffness matrix for element e dependent on x_i :

$$\mathbf{k}_e = \int_V \mathbf{B}^{\mathrm{T}}\mathbf{E}\mathbf{B}\left|\mathbf{J}\right| dV$$

$$\mathbf{k}_{e,x_i} = \int_V \left[\mathbf{B}^T,_{x_i} \mathbf{E}\mathbf{B}\left|\mathbf{J}\right| + \mathbf{B}^T\mathbf{E}\mathbf{B},_{x_i}\left|\mathbf{J}\right| + \mathbf{B}^T\mathbf{E}\mathbf{B}\left(\left|\mathbf{J}\right|\right),_{x_i} \right] dV$$

Example for bar element (direct derivative):

$k_e = EA/L[\ ]$; $x_i \equiv A$; $k_{e,A} = E/L[\ ]$; (code: e.g. $A = 1$)

Caution : FDM schemes may include error in the procedure unless compensated.

3. Sensitivity in dynamic problems

In the dynamic problems, the central task is to determine natural frequencies (eigenvalues) and corresponding modes of vibration (eigenvectors) of the following problem:

$$\mathbf{K}\mathbf{u} = \omega^2 \mathbf{M}\mathbf{u}$$

Introducing $\lambda_i = \omega^2$ for natural frequencies and $\boldsymbol{\Lambda}$ for modes of vibration which satisfy the relation:

$$\det\left(\mathbf{K} - \lambda_k \mathbf{M}\right) = 0 \rightarrow \lambda_k;\ \text{k=1,2…dof.}$$

$$\left(\mathbf{K} - \lambda_k \mathbf{M}\right)\boldsymbol{\Lambda}_k = 0 \rightarrow \boldsymbol{\Lambda}_k;$$

$$\boldsymbol{\Lambda}_k^T\left(\frac{\partial \mathbf{K}}{\partial x_i} - \lambda_k \frac{\partial \mathbf{M}}{\partial x_i}\right)\boldsymbol{\Lambda}_k - \frac{\partial \lambda_k}{\partial x_i}\underbrace{\boldsymbol{\Lambda}_k^T \mathbf{M} \boldsymbol{\Lambda}_k}_{1} + \underbrace{\boldsymbol{\Lambda}_k^T\left(\mathbf{K} - \lambda_k \mathbf{M}\right)}_{=0}\frac{\partial \boldsymbol{\Lambda}_k}{\partial x_i} = 0$$

normalization see above

$$\frac{\partial \lambda_k}{\partial x_i} = \boldsymbol{\Lambda}_k^T\left(\frac{\partial \mathbf{K}}{\partial x_i} - \lambda_k \mathbf{K}\frac{\partial \mathbf{M}}{\partial x_i}\right)\boldsymbol{\Lambda}_k$$

4. Sensitivity Analysis by Fractional Factorial Experiments

In order to examine the sensitivity of the preferred design to changes in some of the parameters, a sensitivity analysis should be performed.

In this way, it can be established which parameters have the greatest effect on the criteria and must therefore be carefully controlled.

Sensitivity Analysis via FFE

- A metric developed by Taguchi (1991) is the ratio of the mean of the attribute value (μ), resulting from the design variables values, to the variation resulting from uncertain parameter values measured via standard deviation (σ).

$$SN = 10\log(\mu^2/\sigma^2) = 10\,(\log\mu^2 - \log\sigma^2)$$

- In fact, it is the ratio of predictability versus unpredictability.

- For the 'nominal is the best' type of attribute, the signal-to-noise ratio reads:

$$SN_N = 10\log\left(\frac{\mu^2}{\sigma^2}\right)$$

Sensitivity Analysis via FFE

- 'More is better' (strength, safety, speed):

$$SN_L = -10\log\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^{\,2}}\right)$$

- 'Less is better' (weight, cost, risk):

$$SN_S = -10\log\left(\frac{1}{n}\sum_{i=1}^{n}y_i^{\,2}\right)$$

- The ratio is based on analysis of variance and is developed from quadratic loss function. It is used to determine the relative importance of the various effects.

Sensitivity Analysis via FFE

- However, an incoherent answer can also be obtained since the ratio is function of the attribute's mean and its variance, i.e. sometimes conflicting measures of position and dispersion.

- It may be argued that separate use of mean and variance is advantageous in search for the 'design window' where design is robust and effectively controlled by design variables.

- Taguchi also deserves credit for suggesting that experimental design techniques such as fractional factorial designs (FFD) could be used as formal part of engineering design methodology.

- Interactions among design variables and design parameters can also be investigated, although it is argued that use of SN largely eliminates this need.

Sensitivity Analysis via FFE

- The method also prefers usage of three levels of factors to estimate the curvature of performance function (attribute) with respect to the factor variability, although obtained quadric is incomplete.

- The method often proves very fruitful in application thus confirming the power of FFD concept in quality engineering.

- However the question remains if the two level approach with mean point can lead to fewer experiments and whether surface fitting methods can give superior results (Montgomery, 1991).

Sensitivity Analysis via FFE

Embedding of sensitivity analysis into design process can be done in different ways, e.g:

1. Post-processing of non-dominated solutions with respect to the robustness attribute.
2. Direct inclusion of robustness attribute in the process of generation of non-dominated designs.
3. Combination of 1. and 2. by calculating the robustness attribute value only in the final cycles of sequential generation of non-dominated hyper-surface by simultaneously calculating attribute and its measure of robustness.

Application of orthogonal arrays in these cycles, as described simplifies the calculation.

Sensitivity Analysis via FFE

For the third approach, the sensitivity calculation applied is summarized in the sequel:

- Choose parameter levels broad enough to include realistic range of possibilities and be not mislead by some local 'plateau' of attribute function. Levels should also be narrow enough to avoid impossible designs.

- Define variable levels for current 'minicube' cycle.

- Perform global experiments (designs) using orthogonal arrays to generate design variables.

Sensitivity Analysis via FFE

- Perform local experiments for each global experiment to calculate measure of robustness for all affected attributes in **y**.

$$\sigma^2 = \frac{\sum_{k=1}^{N} y_k^2 - \frac{\left(\sum_{k=1}^{N} y_k\right)}{N}}{N-1}$$ where N= number of experiments,

$$\mu_e^2 = \frac{N.M^2 - \sigma^2}{N}$$ where $M = \frac{1}{N}\sum_{k=1}^{N} y_k$,

$$SN = 10\log(\mu_e^2 / \sigma^2)$$

(Small size of sample is taken into account when calculating estimates of standard deviation squared and approximate mean square of attribute.)

Sensitivity Analysis via FFE

- Calculate level variations of each attribute for each level of design variable to estimate influence of variable on the attribute robustness.
- Calculate compound measure of the current design robustness using:
  (a) first order Taylor expansion of the weighted utility function, if any;
  (b) maximal achieved robustness measure of all design attributes.

  Note that (a) is possible only in metric space, while (b) is possible for normalized attributes (via membership grade approach) in affine space and therefore is practical in design generation phase.

- Eliminate dominated designs with respect to all design attributes including robustness attribute.

Sensitivity Analysis via FFE

	Intop					1	1	1	2	2	2	3	3	3			
	Bperc					1	2	3	1	2	3	1	2	3			
	Nloan					1	2	3	2	3	1	3	1	2			
Trial	L_{pp}	B_x	T_x	C_p	C_x	Membership grade for a combination of variables and parameters levels									σ	μ_e	S/N
1	1	1	1	1	1	0.868	0.907	0.903	0.941	0.892	0.745	0.945	0.839	0.823	0.064	0.874	22.76
2	1	1	2	2	2	0.620	0.645	0.660	0.710	0.648	0.513	0.715	0.594	0.578	0.064	0.631	19.86
3	1	1	3	3	3	0.406	0.422	0.433	0.468	0.424	0.337	0.472	0.388	0.378	0.043	0.414	19.72
4	1	2	1	2	2	0.783	0.807	0.822	0.867	0.810	0.665	0.872	0.755	0.739	0.065	0.791	21.70
5	1	2	2	3	3	0.525	0.546	0.559	0.601	0.548	0.438	0.606	0.503	0.491	0.053	0.535	20.04
6	1	2	3	1	1	0.795	0.819	0.834	0.878	0.822	0.677	0.883	0.767	0.751	0.065	0.803	21.87
7	1	3	1	3	3	0.715	0.738	0.753	0.797	0.741	0.608	0.802	0.689	0.674	0.061	0.724	21.44
8	1	3	2	1	1	0.944	0.959	0.967	0.987	0.961	0.854	0.989	0.926	0.914	0.042	0.944	27.01
9	1	3	3	2	2	0.743	0.766	0.781	0.825	0.769	0.634	0.830	0.717	0.702	0.062	0.752	21.72
10	2	1	1	2	3	0.612	0.636	0.652	0.700	0.640	0.508	0.705	0.587	0.572	0.062	0.623	19.99
11	2	1	2	3	1	0.623	0.647	0.662	0.711	0.650	0.517	0.716	0.597	0.582	0.063	0.634	20.04
12	2	1	3	1	2	0.643	0.668	0.683	0.732	0.671	0.535	0.738	0.616	0.601	0.064	0.654	20.14
13	2	2	1	3	1	0.782	0.806	0.820	0.865	0.809	0.666	0.870	0.754	0.738	0.064	0.790	21.83
14	2	2	2	1	2	0.789	0.813	0.828	0.871	0.816	0.673	0.876	0.762	0.746	0.064	0.797	21.93
15	2	2	3	2	3	0.540	0.560	0.574	0.616	0.563	0.451	0.621	0.518	0.505	0.054	0.549	20.20
16	2	3	1	1	2	0.954	0.967	0.974	0.991	0.968	0.869	0.992	0.937	0.926	0.039	0.953	27.86
17	2	3	2	2	3	0.727	0.750	0.764	0.808	0.753	0.621	0.812	0.701	0.687	0.060	0.736	21.73
18	2	3	3	3	1	0.750	0.773	0.787	0.830	0.776	0.643	0.835	0.725	0.710	0.060	0.759	21.98
19	3	1	1	3	2	0.623	0.647	0.663	0.711	0.651	0.519	0.716	0.597	0.583	0.063	0.634	20.12
20	3	1	2	1	3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.00
21	3	1	3	2	1	0.633	0.657	0.673	0.721	0.661	0.527	0.726	0.607	0.592	0.063	0.644	20.17
22	3	2	1	1	3	0.769	0.793	0.807	0.852	0.796	0.655	0.857	0.741	0.726	0.064	0.777	21.73
23	3	2	2	2	1	0.777	0.801	0.815	0.860	0.804	0.662	0.864	0.750	0.734	0.064	0.785	21.83
24	3	2	3	3	2	0.547	0.568	0.581	0.624	0.571	0.458	0.628	0.525	0.512	0.054	0.557	20.29
25	3	3	1	2	1	0.945	0.959	0.967	0.987	0.961	0.858	0.988	0.927	0.916	0.041	0.945	27.31
26	3	3	2	3	2	0.733	0.756	0.770	0.813	0.758	0.628	0.818	0.708	0.693	0.060	0.741	21.85
27	3	3	3	1	3	0.734	0.757	0.771	0.814	0.760	0.629	0.819	0.709	0.695	0.060	0.743	21.87

Fractional factorial design around design with minimum L_∞

Example of RoPax ship RFR for preferred design L_∞