L4b ROPAX RB DESIGN

CONTENS

- ANALISYS MODULES
- SINTHESYS MODULES
- APPLICATION TO ROPAX EXAMPLE
- DECISION SUPPORT ENVIRONMENT

CONCLUSIONS

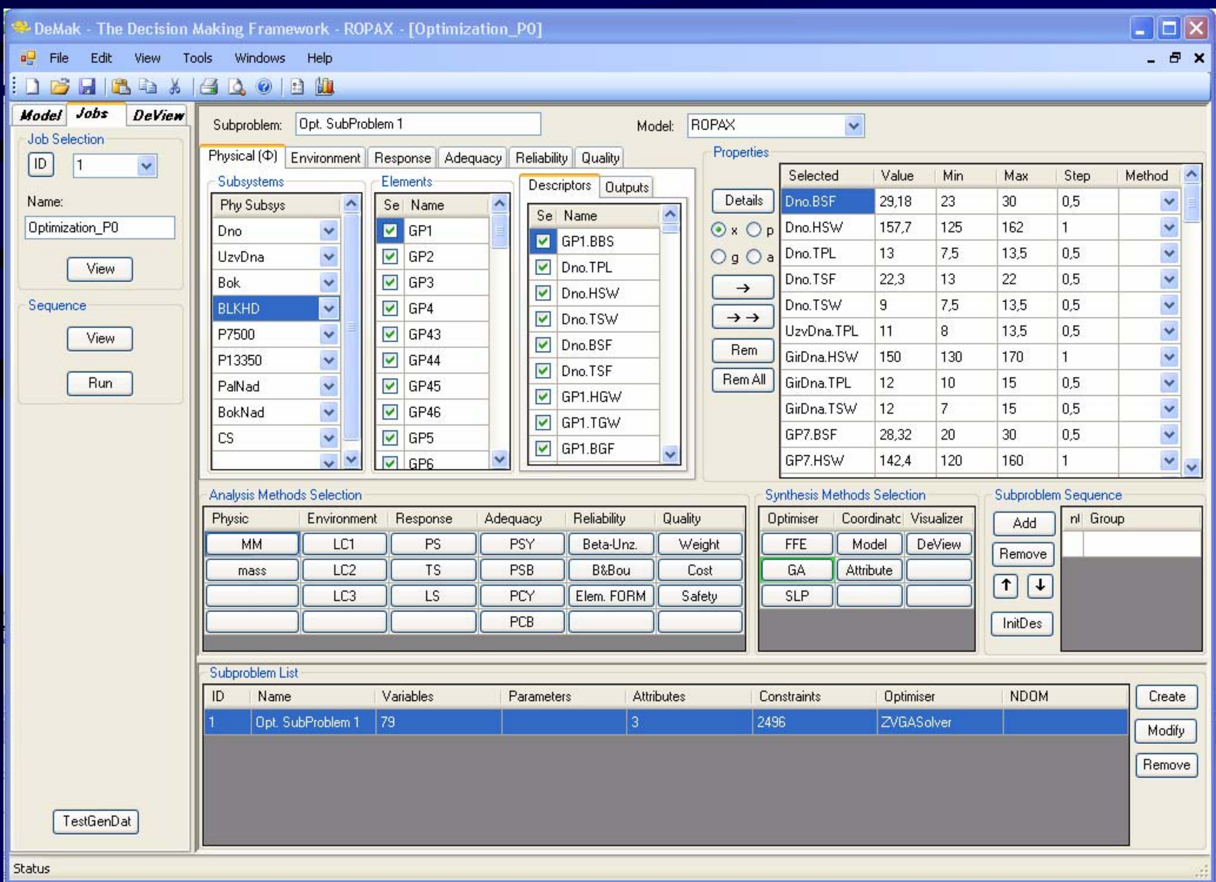
ANALISYS MODULES

ANALYSIS MODELS	OCTOPUS ANALYZER MODULES
Physical **(Φ)**	**FEM STRUCTURAL MODELER** **MIND – generator of minimal dimensions**
Environment **(ε)**	**OCTLOAD -** load model
Response **(ρ-1)**	**LTOR-** primary strength fields (warping displac.; normal/shear stresses)
Response **(ρ-2)**	**TOKV** -secondary strength fields: transverse and lateral displacements, stresses
Adequacy / feasibility **(α-1)**	**EPAN** – library of stiffened panel and girder ultimate strength & serviceability criteria. (FATCS – Rules fatigue calculation-Level 1)
Adequacy (α-2)	**LUSA** – Ultimate longitudinal strength module
Reliability **(π-1,2)**	**US-3** reliability calculation of element and system failure probability (level 1-3, mechanism.) **SENCOR – sensitivity** to correlation.
Quality **(Ω-1 to 8)**	**WST / INC - cost/weight** **DCLV - ultimate vertical bending moment** **DCLT- ultimate racking load** **SSR / SCR - reliability measures** **ICM / TSN - robustness measures**

SYNTHESYS MODULES

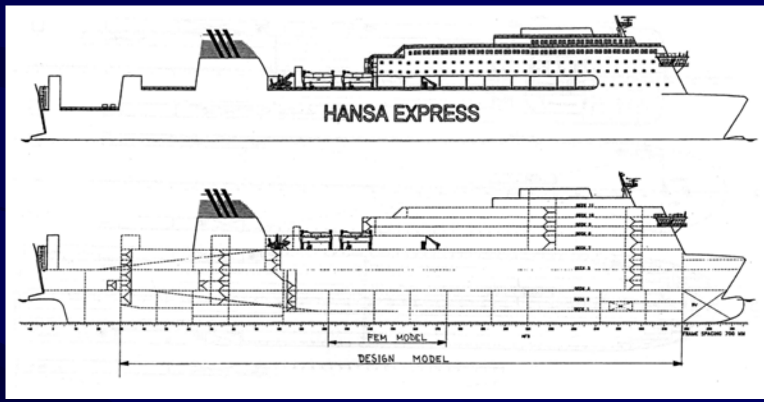
SYNTHESIS MODELS	OCTOPUS DESIGNER MODULES
Problem definition (Δ)	C# shell: **SYNCHRO** – decision support problem definition, selection of analysis and synthesis methods. **Auxiliary modules:** **CAPLAN – control of Pareto surface generation** **LINC – definition of feasible subspace based on subset of linear/linearized constraints**
Problem solution (Σ)	DeMak optimization solvers: **MONTE – multilevel multi criteria evolution strategy** **FFE – Fractional Factorial Experiments** **CALMOP - SLP cross section optimizer** **MOGA - Multi objective GA** **DOMINO – Pareto frontier filter** **MINIS – subspace size controller** **HYBRID – combination solver-sequencer**
Problem graphics and interactivity (Γ)	MAESTRO Graphic Environment De View C# Environment **Design selection modules in metric space:** **GOAL- interactive goal input** **SAATY - inter-attribute preferences** **FUZZY - intra-attribute preferences** **COREL - statistical analysis of results**

OCTOPUS - DECISION MAKING FRAMEWORK

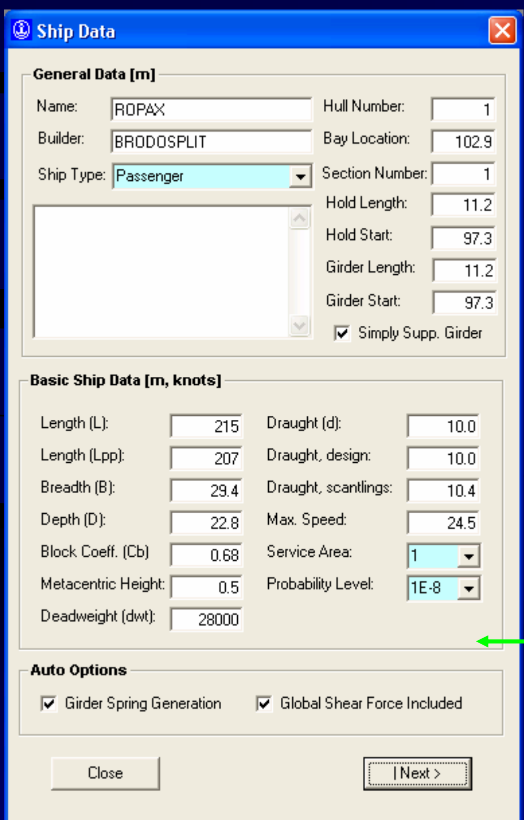


EXAMPLE OF APPLICATION: ROPAX SHIP

- **The ship's** main dimensions:
  - Loa=221.2m;
  - Lanes=3500m;
  - B=29m;
  - D=16.4m;
  - Tsc=7.4m;

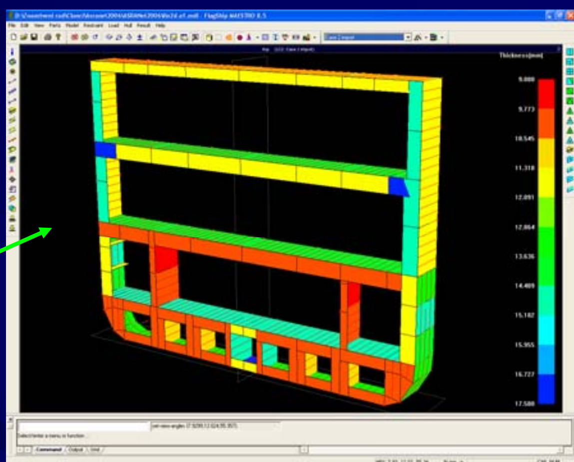


PHYSICAL (Φ): - FEM STRUCTURAL MODELER, - MINIMAL DIMENSIONS MODULE

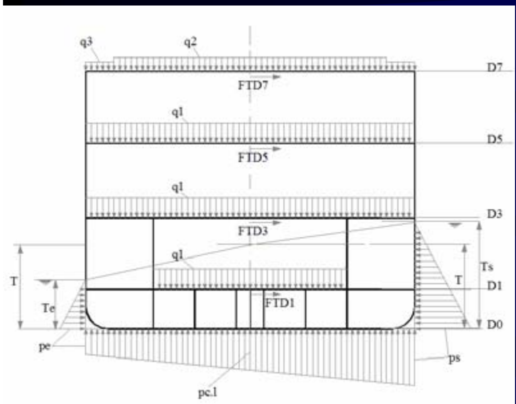


- MAESTRO MODELER used to define 2.5D FEM model with different cross-sections (web-frame, bulkhead).
- MIND (minimal dimensions definition from Class. Society Rules-DNV).

RoPax



ENVIRONMENT (ε): - OCTLOAD



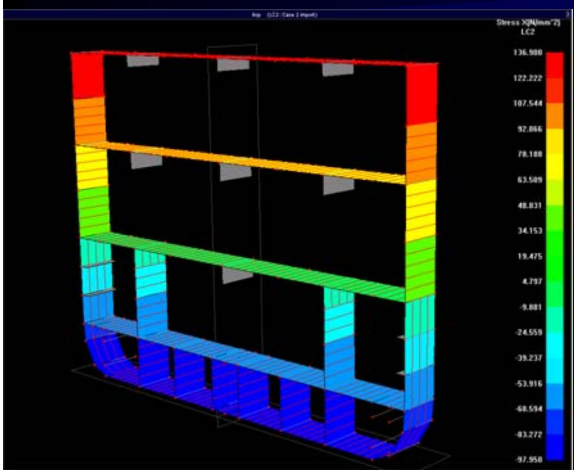
LC 6 and 7

RoPax

- Class. Society Loads - DNV (Note: CRS and IACS -CSR are generated automatically - CREST software).
- Designer given loads from seakeeping analysis (3D Hydro model) are optional input.

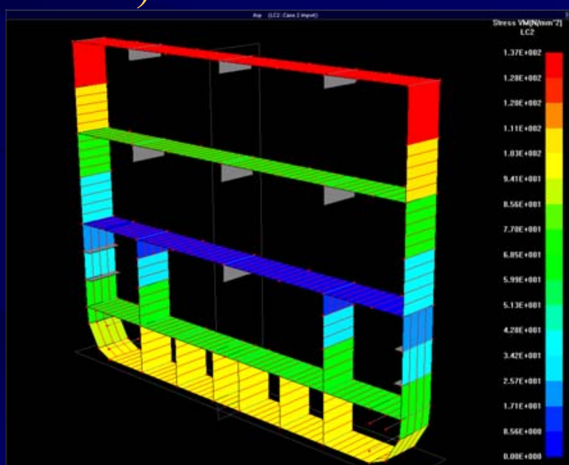
LC	DESCRIPTION
1-SAGG	Full load on decks + dyn. / Scantling draught
2-HOGG	Full load on decks + dyn. / Scantling draught
3-SAGG	Full load on decks except D1 + dyn. / T- scantling
4-HOGG	Full load on decks except D1 + dyn. / T- scantling
5-HOGG	Ballast condition /Draught 5.8 m
6-SAGG	Full load on decks + dyn. / Heeled condition
7-HOGG	Full load on decks + dyn. / Heeled condition

RESPONSE (ρ -1): - LTOR



LC 2 - σ_x

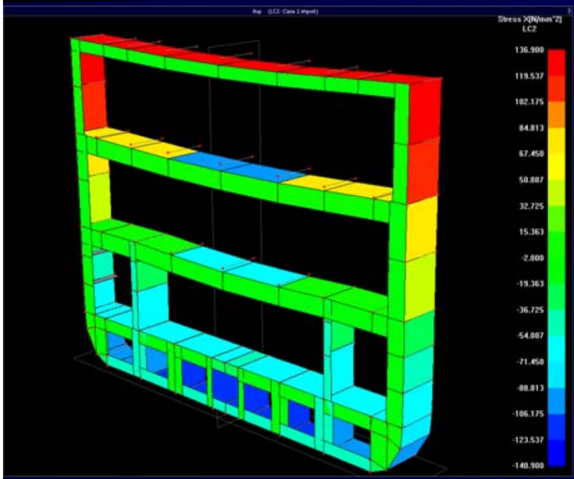
- Primary strength fields
  - Warping displ.; normal/shear stresses
  - Extended beam theory (cross section warping fields via FEM in vertical / horizontal bending and warping torsion)



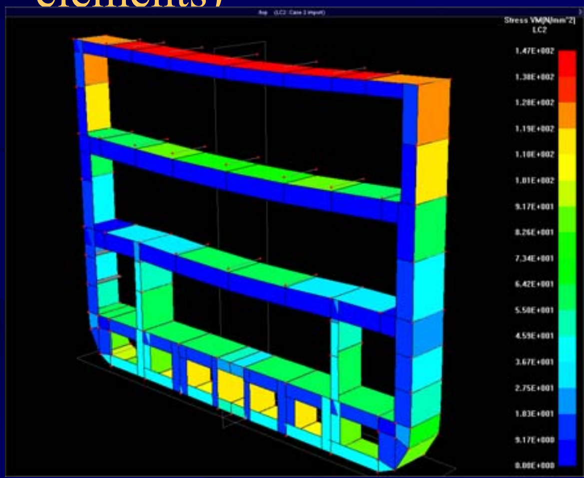
LC 2 - σ_{VM}

RoPax

RESPONSE (ρ -2): - TOKV



- Secondary strength fields:
  - transverse and lateral displ.; stresses
  - FEM analysis of web-frame and bulkhead (beam element with rigid ends; stiffened shell 8-node macro-elements)



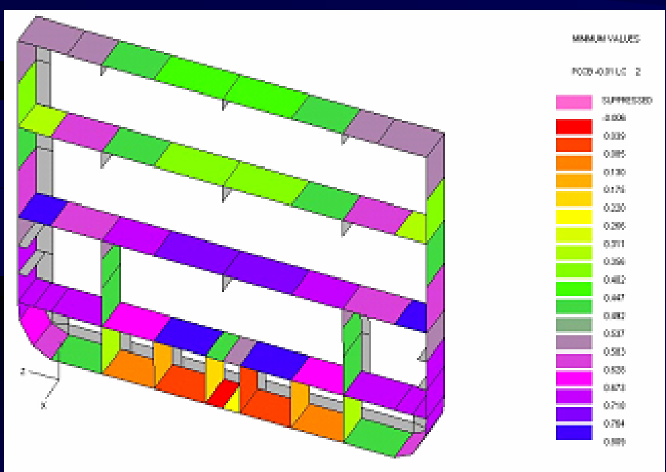
LC 2 - σ_x

LC 2 - σ_{VM}

RoPax

ADEQUACY (α -1) : - EPAN

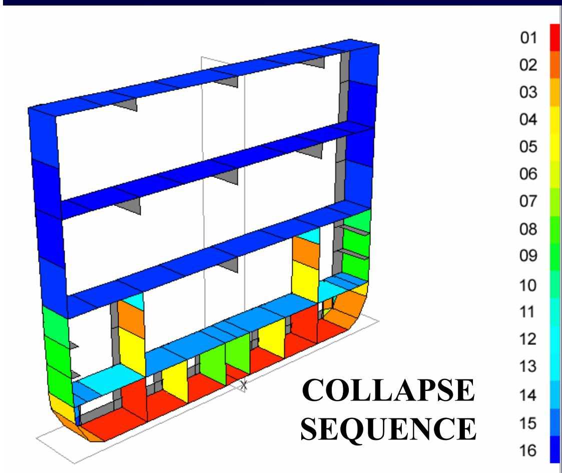
- Library of stiffened panel and girder ultimate strength & serviceability criteria
  - Calculation of macroelement feasibility based on super-position of response fields ρ-1, ρ-2 (FEM); ρ-3 (analytical) and using the library of analytical safety criteria



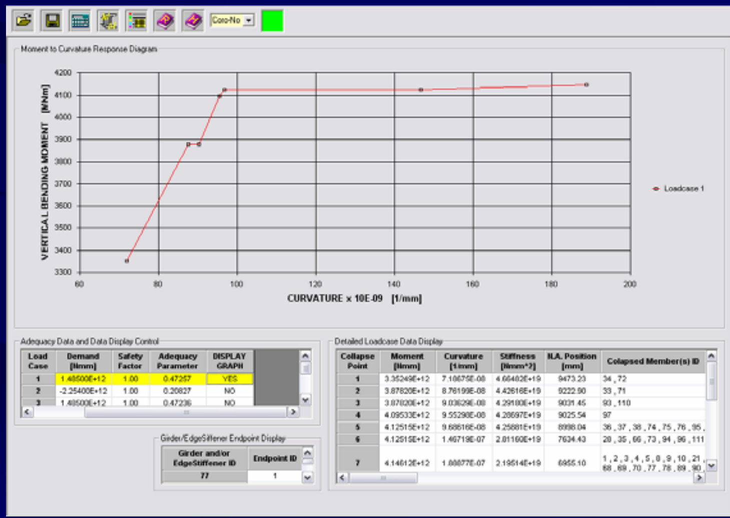
$$-1 \le g(x) = \frac{C - SF \cdot D}{C + SF \cdot D} \le 1,$$

NAME	CRITERIA DESCRIPTION - PLATE
PCMY	**Panel Collapse Membrane Yield (Von Misses)**
PYLS	**Panel Yield Longitudinal Strength**
PCAPS	**Panel Collapse Arched Plate Yield**
PCAPT	**Panel Collapse Arched Plate Shear**
PFLB	**Panel Failure. Local Buckling**
PCES	**Panel Collapse Edge Shear**
S-UCS	**SLS, Uniaxial Compressive Stress**
U-UCS	**ULS, Uniaxial Compressive Stress**
S-ES	**SLS, Edge Shear**
U-ES	**ULS, Edge Shear**
S-ULL	**SLS, Uniform Lateral Load**
U-ULL	**ULS, Uniform Lateral Load**
....	**……………………..**

ADEQUACY (α-2) : - LUSA



- Ultimate longitudinal strength
  - Incremental ultimate strength analysis of cross-section using IACS and extended Hughes/Adamchak procedures



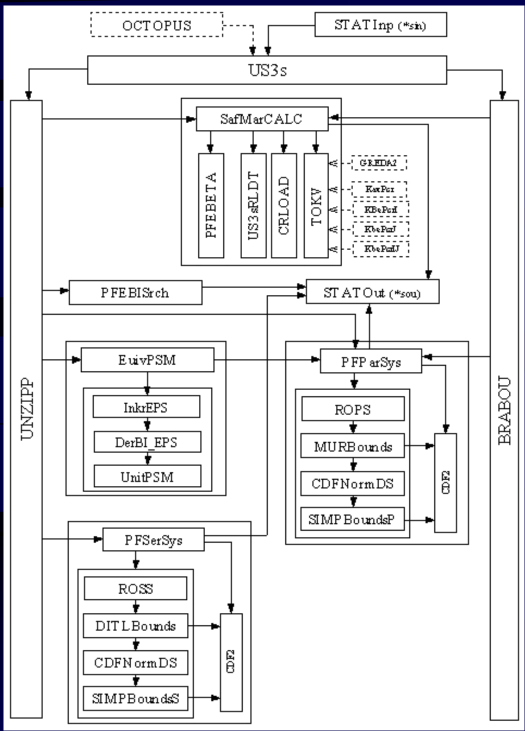
RoPax

RELIABILITY (π-1): - US3

- Element and system failure probability (level 1-3, mechanism)
  - (1) FORM approach to panel reliability
  - (2) β -unzipping method for system probability of failure

  - Probabilistically dominant collapse scenarios are selected from the (large) set of potential collapse scenarios at the first, second, third and mechanism level.
  - The system reliability measure at third level (RM-3) was found sufficient for the optimization (design) purpose.
  - RM-3 is modeled as a series system of all identified, probabilistically dominant collapse scenarios.
  - Structural redundancy can be also assessed from the most dominant failure scenarios

RELIABILITY (π-1): - US3



Flow – chart

OCTOPUS-US3s module description	Module
Reliability measure calculation using the β-unzipping procedure	UNZIPP
Statistical input (load and resistance variables, correlation matrix for loads and resistance variables)	STATInp
Automatic generation of potential failure element model. Automatic generation of potential collapse scenarios. Identification of probabilistically dominant collapse scenarios.	PFEBISrch
Calculation of safety margin for potential failure elements	SafMarCAL C
Equivalent safety margin calculation for each collapse scenario.	EquivPSM
Reliability measure calculation for each identified collapse scenario. Murotsu bounds, Dunnet-Sobel method, Simple bounds)	PFParSys
System reliability measure calculation of the structure - modeled as serial system of identified, probabilistically dominant collapse scenarios.(Ditlevsen bounds, Dunnet-Sobel method, Simple bounds)	PFSerSys

Modules

RELIABILITY (π-2): - SENCOR

- Sensitivity to correlation of input variables (based on Nataf model)

- $\mathbf{B}_{//R} = [B_{i,km}]$ - sensitivities of failure mode safety indices $\boldsymbol{\beta}_i$ to elements of correlation matrix **R** $[\rho_{km}]$
- $\mathbf{P}_{//R} = [P_{i,km}]$ - sensitivities of modal failure probabilities $P_i \equiv P_{fi}$)
- $\mathbf{G}_{//R} = [G_{ij,km}]$ - sensitivities of bimodal correlation coefficients γ_{ij}
- $\mathbf{H}_{//R} = [H_{ij,km}]$ - sensitivities of joint failure probabilities P_{ij} (modes i&j)
- $\mathbf{P^B}_{//R} = [P^U{}_{,km}]$ - sensitivities of failure probability bounds (eg. Ditlevsen upper bound)
- $\mathbf{B^G}_{//R} = [B^G{}_{,km}]$ - sensitivities of generalized safety index: $\beta_G = -\Phi^{-1}(P^B)$
- The expressions for all important sensitivity matrices with respect to modified correlation matrix **R'** [3] are given in a very simple.
- For Ditlevsen's upper bound the sensitivity matrix and the safety index sensitivity matrix read:

$$\mathbf{P}^{BUseries}_{//R} = \sum_{i=1}^{n_f} \mathbf{P}^{<i>}_{//R} - \sum_{i=2}^{n_f} \left(\mathbf{H}^{<i\&j>}_{//R} \middle| \max_{j<i} P_{ij} \right)$$

$$\mathbf{B}^{G}_{//R} = -\Phi^{-1}\left(\mathbf{P}^{BUseries}_{//R}\right)$$

QUALITY (Ω): DESIGN ATTRIBUTES

- INC / WST - cost/weight modules
  - Minimal initial cost
  - Minimal struct. weight =max. DWT increase
- DCLV - ultimate vertical bending moment
  - Calculations using LUSA
- DCLT- ultimate racking load
  - (Deterministic calculation using US-3 analysis module)
- SSR / SCR - reliability measures
  - Upp. Ditlevsen bound of panel failure/ racking failure prob
- ICM / TSN - robustness measures
  - (Information context measure / Taguchi S/N ratio via FFE).

CONCEPT DESIGN OF ROPAX :

- For ship redesign the Yard defined the design objectives:
  - minimal mass and cost,
  - minimal ship height D,
  - maximal safety measure
- Prototype geometry and topology, design load cases, design parameters, design variables and constraints were to be in accordance with the Yard's practice and DNV Rules for DC.

CONCEPT DESIGN OF ROPAX :

- Basic to the procedure is the treatment of structural adequacy as design quality measures (attributes).
- Those quality measures are most instructive if based on the system's ultimate strength (ultimate capacity)
- In the described procedure they are:
  - the ultimate bending moment in sagging / hogging,
  - the system reliability measure for racking (including nonlinear frame racking analysis)

thus measuring effectively the quality and feasibility of the entire design variant.

Design sequence

	Step	Task	Method	Module*
Prototype response analysis	1a	Rule load analysis	DNV	OCTLOAD
	1b	Seakeeping load analysis	3D- panel	BV HydroStar
	2a	Structural response and adequacy analysis	2.5-D FEM	LTOR-TOKV-EPAN
	2b	Primary ultimate strength analysis	Nonlinear analysis	LUSA+2a
	2c	Deterministic racking analysis	2-D FEM	TOKV-EPAN
	3a	Probabilistic a. of primary response	M_{SW}, M_W, M_{ULT}	CALREL / SORM+2b
	3b	Probabilistic a. of racking response	β-unzipping	US3+2c
Concept design	4a	**Reliability based concept optim.**	**OA (L27) designs**	**DEMAK / FFE+2b+3b**
	4b	**Filtering of Pareto prototypes**	**$p_{f\text{-}rack}$ - mass - $M_{long\text{-}ult}$**	**DEMAK (DOMINO)**
	4c	**Selection of preferred designs**	**Value function**	**DEMAK-DEVIEW**
	5	Deterministic optimization of preferred designs	Hybrid optimizer	DEMAK / SLP+FFE+ +2abc
	6	Reliability based re-optimization of optimal design	OA (L27) designs	DEMAK / FFE+3b
Preliminary design	7a	**Structural analysis and optimization**	**3-D FEM +SLP +DEMAK**	**MAESTRO**
	7b	Probabilistic analysis of opt. design racking	β-unzipping	US3+2c
	7c	Robustness analysis	Taguchi S/N Ratio	ROBUST

* see Table 1 and Figure 1.

PROTOTYPE:SAFETY ANALYSIS

Prototype deterministic safety analysis showed that prototype failed in 35 criteria w.r.t DNV Rules (out of 8820 checks for 7 LCs) in:

- double bottom (stiff. panels/ frames $g_{FCPB,}$= -0.268)
- tank-side (st. panels e.g. $g_{U\text{-}BCAES,min}$= -0.172)
- deck5-middle (st. panel e.g. $g_{PFLB,min}$=-0.243)

- Ultimate bending moment-LC1(sagg)=3.93 106 kNm LC2 (hogg)=3.18 106 kNm (bottom collapse in compression-see above).
- Identified failed elements were non-optimally strengthened (mass increased 1.2%; strong prototype ■)

- System failure probability (Ditlevsen upper bound) for the 45 identified relevant (level-3) failure scenarios was: p_f=0.101·10^{-6}; β_G=5.198 showing the existence of considerable safety margin

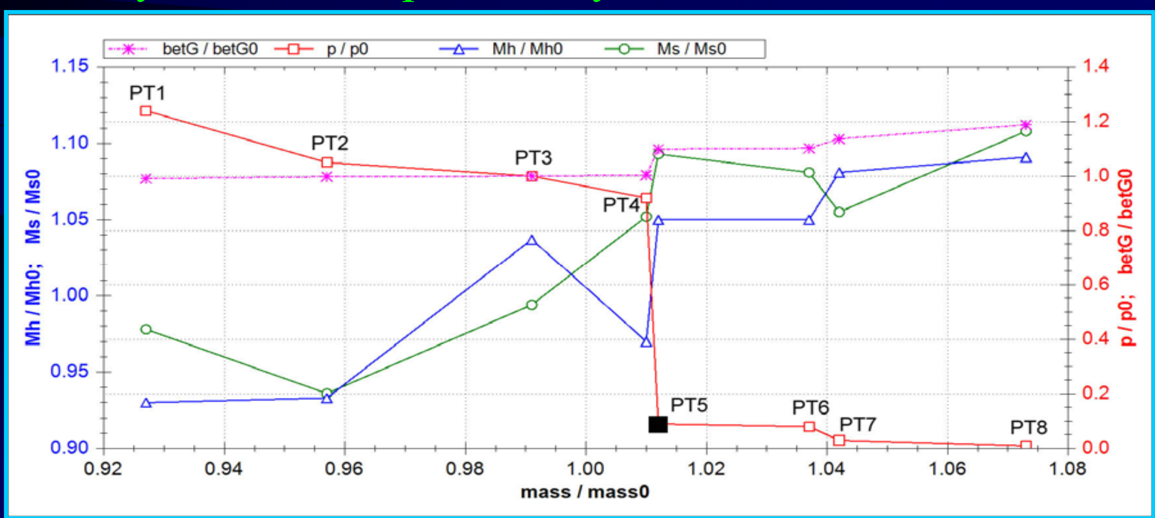
CONCEPT DESIGN OF ROPAX

- Concept exploration included generation of designs via orthogonal arrays based upon Latin squares (FFE). Concept design model included 36 design variables. Levels were defined via variation of plate and frame scantlings/ thicknesses
- Regarding safety measure, for variant relative comparisons, the COV of marginal distributions for all load components were taken uniformly as 15% and 5% for capabilities (in this example).
- Through the dominance filtering, the eight non-dominated designs were generated. The dominant failure scenarios were identified.

CONCEPT DESIGN : PREFERED DESIGNS

Attributes normalized to the original prototype values:

- Mass,
- $M_{ulthogg}$,
- $M_{ultsagg}$,
- System failure probability



RESULTS :

- The most probable racking failure scenario included failures at deck 3 (close or at tank-side), followed by the bilge structure collapse.
- For further increment in mass of 3% (point PT7) the probability of failure could be further improved: $p_f=0.374 \cdot 10^{-7}$ / $\beta_G=5.380$.

RESULTS :

- After strengthening the side and bottom structures and reducing the rest of scantlings, the system failure probability was the acceptable $p_f = p_0 = 0.118 \cdot 10^{-5}$/ $\beta_G = 4.72$, also with acceptable decrease in ultimate bending moments and with solved local prototype problems. Total mass reduction was -2.2% (PT3: PT5)
- For permitted scantling reduction, the system failure probability increased to $0.1393 \cdot 10^{-5}$ / $\beta_G=4.69$.
- **Weight was reduced by 7.3%** with decrease in ult. bending moment and with solved prototype problems (PT1).

PRELIMINARY DESIGN:

- Elaborate concept/preliminary optimization of the same prototype (with nv = 264, nconstr = 56416) was performed with 3D FEM partial model. It has shown that significant reduction of up to 9.5% in steel mass (**560 t of extra DWT**) can indeed be achieved with satisfied DNV Rules.
- Preliminary optimization has corroborated the usefulness of the concept design results presented here. Note: both of these optimizations started from the same prototype.

PRELIMINARY DESIGN:

- The last step of concept design is the repetition of the described concept exploration step but centered around selected optimal design variants (e.g. PT1, PT3).
- The subjective reasoning of head designer and his prejudices with respect to safety versus cost are part of the yard/owner policy. Extensive investigation in those aspects of the problem is currently being underway for EU and domestic projects.

CONCLUSIONS ON CONCEPT DESIGN:

- **The reliability based design procedure for concept design phase, using the developed interactive design environment, can give a rational initiative for the design improvement using safety as attribute.**

- **It is based on the powerful global feasibility and reliability measures for ultimate primary / secondary strength of complex multi-deck ships.**

- **Only relative comparisons of safety attributes are needed in design filtering, resolving thus the problem of required accuracy of the analysis methods.**

- **Safety as an objective, not only as a constraint, is a way towards the true meaning of the design paradigm: 'safety versus cost' with two competing objectives.**

SENCOR SENSITIVITY ANALYSIS (not part of the course)

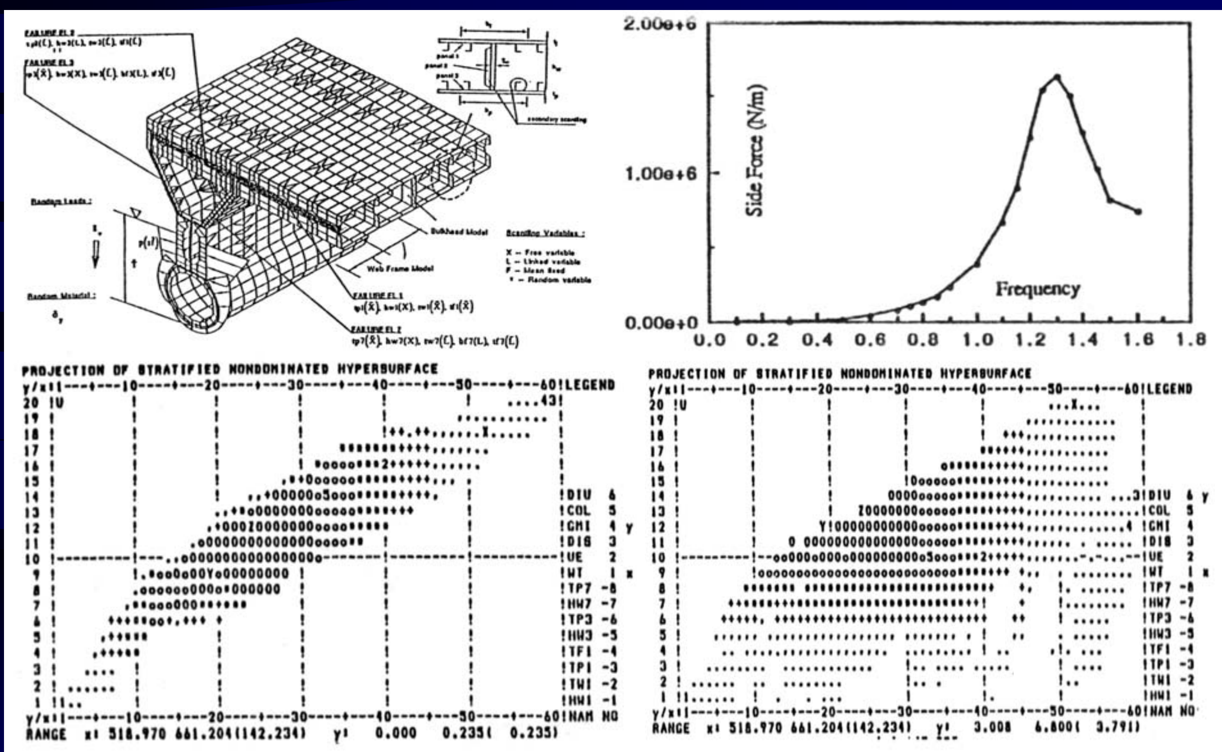
CONTENTS

1. INTRODUCTION

SWATH Reliability Based Design

Concept Design Model

RAO of side force(beam sea)



Mass vs safety factor

Mass vs reliability

MODULE	OCTOPUS
(4) FEASIBILITY CALCULATION (Normalized Safety Factor)	Calculation of macroelement feasibility using library of safety criteria in program PANEL (C – capability; D – demand)
(5) **RELIABILITY CALCULATION**	**FORM approach to panel reliability.** **Upper Dietlevsen bound as design attribute**
(6) **DECISION SUPPORT PROBLEM DEFINITION** (interactive)	Constraints: User given Minimal dimensions Library of criteria (see 4) Objectives: Minimal weight, Minimal cost **Maximal safety,** Maximal collapse load
(7a, b,c) **OPTIMIZATION METHOD**	Decision making procedure using a) Global MODM program GLO b) Local MADM module LOC c) Coordination module GAZ
(8a,b,c) PRESENTATION OF RESULTS	a) VB Environment, b) Program MG, c) DeVIEW graphic tool

CORRELATION COEFFICIENTS

- **Wave load components :**

$$\rho_{ij} = \frac{1}{\sigma_i \sigma_j} \int_0^{\infty} H_i(\omega)\, H_j^*(\omega)\, S_{xx}(\omega)\, d\omega$$

where:
$H_i(\omega)$ - system functions,
$S_{xx}(\omega)$ - input spectra,
σ_i and σ_j - corresponding standard deviations

- **Capability components. Exponentially decaying [Handa, Anderson, 1987]:**

$$\rho_{ij} = -\exp\left(\frac{\kappa \Delta x}{L}\right)$$

where:
Δx - distance between the elements of the length L
κ - defined from experiments

- **Structure in service,corrosion rates in neighbouring corroded elements [Guedes Soares, 1997]**

$$\rho_{ij} = \left[1 - \left|\frac{\sqrt{y_i^2 + z_i^2} - \sqrt{y_j^2 + z_j^2}}{\sqrt{y_{max}^2 + z_{max}^2}}\right|\right]^n$$

2. SENSITIVITY TO CORRELATION MATRICES

- n-tuple or matrix **S=[RM_{ij}]** contains design reliability measures **(RM)** e.g , β_i , P_{fi}, etc.
- **Matrix of derivatives** $S_{,R}$=[∂(RM_{ij}) / ∂ρ_{km}] gives derivatives of **S** w.r.t. elements of correlation matrix **R**
- **Sensitivity matrix** $S_{//R}$ gives sensitivity of **S** w.r.t. elements of correlation matrix **R** as a term-wise product (composition ○) of the matrix of derivatives $S_{,R}$ and the **matrix of multiplicators** $\mathbf{s} = [s_{km}]$:

$$\mathbf{S}_{//R} = [S_{ij//km}] = \mathbf{S}_{,R} \circ \mathbf{s} = [S_{ij,km} s_{km}]$$

- Typical **matrices of multiplicators** **s** are:

$\mathbf{s}^1 = [s_{km} = 1]$ - rate of change (derivative) of reliability measure i.e. $\mathbf{S}_{//R} = \mathbf{S}_{,R}$

$\mathbf{s}^2 = [\Delta\rho_{km}]$ - increment due to perturbation, i.e. most unfavourable deviation

$\mathbf{s}^3 = [\Delta\rho_{km}]/RM_{ij}$ - logarithmic derivative of RM_{ij}

Note : If ρ_{km} are functions of parameters **p**, factors s_{km} would include terms $\partial\rho_{km}(\mathbf{p})/\partial p_i$

BASIC SENSITIVITY MATRICES IN FORM AND SORM :

$\mathbf{B}_{//R} = [B_{i,km}\ s_{km}]$ - sensitivities of failure mode **safety indices** β_i (≡RM_i) to **R**

$\mathbf{P}_{//R} = [P_{i,km}\ s_{km}]$ - sensitivities of **modal failure probabilities** P_i (≡P_{fi})

$\mathbf{G}_{//R} = [G_{ij,km}\ s_{km}]$ - sensitivities of **bimodal correlation coefficients** γ_{ij}

$\mathbf{H}_{//R} = [H_{ij,km}\ s_{km}]$ - sensitivities of **joint failure probabilities** P_{ij} (for modes i & j)

$\mathbf{P}^B_{//R} = [P^U_{,km}\ s_{km}]$ - sensitivities of failure **probability bounds** (eg. Ditlevsen upper bound)

$\mathbf{B}^G_{//R} = [B^G_{,km}\ s_{km}]$ - sensitivities of **generalized safety index** $\beta_G = -\Phi^{-1}(P^B)$

SENSITIVITY ESTIMATES VIA DIFFERENT NORMS $L_p(S_{//R})$:

$L_\infty = \max_{k,m} |S_{ij//km}|$ - identifies the **most influential correlation coefficient** for RM_{ij};

$L_{row} = \max_k \sum_m |S_{ij//km}|$ - the row norm; identifies the **most influential random variable**;

$L_p = (\sum_k \sum_m |S_{ij//km}|^p)^{1/p}$ - gives the **total variability** due to correlation (p=1,2)

BASIC FORMULATION

- Basic random variables............................ $\mathbf{X} = \{x_1 \ldots x_n\}$,
  → marginal or joint PDF and CDF, parameters, correlation matrix $\mathbf{R} = [\rho_{ij}]$.
- Standard normal correlated variables...... $\mathbf{Y} = \{y_1 \ldots y_n\}$;
  → $\mathbf{Y} = T(\mathbf{X})$; PDF= $\phi_n(\mathbf{Y}, \mathbf{R'})$; CDF = $\Phi_n(\mathbf{Y}, \mathbf{R'})$; **modified corr. matrix** $\mathbf{R'} = [\rho'_{ij}]$

  → **Sensitivity of** ρ'_{ij} **to original** ρ_{ij} → $e = [\partial \rho'_{ij} / \partial \rho_{ij}]$ → **Derivative matrix** $\mathbf{S}_{,\mathbf{R}} = \mathbf{S}_{,\mathbf{R'}} \circ \mathbf{e}$

  Nataf model : marginal transformations $F(x_i) = \Phi(y_i)$, $\mathbf{R} \rightarrow \mathbf{R'}$ (Nataf tables)
- Independent standard normal variables... $\mathbf{U} = \{u_1 \ldots u_n\}$

  → $\mathbf{Y} = \mathbf{AU}$; PDF= $\phi_n(\mathbf{U},\mathbf{I})$; CDF= $\Phi_n(\mathbf{U},\mathbf{I})$;
  → $\mathbf{A}$ from $\mathbf{R'} = \mathbf{AA}^T$;
  Spectral decomposition $\mathbf{A} = \mathbf{V}\boldsymbol{\Lambda}^{1/2}$: $\boldsymbol{\Lambda}$ = variances (eigenvalues) ;
  (Cholesky decomposition $\mathbf{A} = \mathbf{L}$) $\mathbf{V}$ = principal directions (eigenvectors)
- **Failure probability :**

$$P_f = \iint_{\Omega(\mathbf{X})} f(\mathbf{X},\mathbf{R})\,d\mathbf{X} = \iint_{\Omega(\mathbf{Y})} \phi_n(\mathbf{Y},\mathbf{R}')\,d\mathbf{Y} = \iint_{\Omega(\mathbf{U})} \phi_n(\mathbf{U},\mathbf{I})\,d\mathbf{U}$$

$$\Omega(\mathbf{X}) = \left\{ \mathbf{X} \;\middle|\; g_i(\mathbf{X}) \le 0, i = 1 \ldots nf \right\} \qquad d\mathbf{X} = \prod_n dx_i$$

3. APPLICATION

EXAMPLE 1 PROBLEM DEFINITION :

- random variables x_i (i=1,..7);
- marginal distributions: Weibull (x_1-x_5) and uniform (x_6-x_7);
- prescribed means μ, standard deviations σ and correlation matrix $\mathbf{R}$;
- $\mathbf{R'}$ from Nataf model tables
- series system; linear failure surfaces $g_j < 0$ (j=1, 5) :

$$g(x) := \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & -5 & 0 \\ 1 & 0 & 2 & 2 & 1 & -5 & -5 \\ 0 & 1 & 2 & 1 & 0 & 0 & -5 \\ 1 & 2 & 1 & 0 & 0 & -4 & 0 \\ 0 & 1 & 1 & 2 & 0 & -4 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} \quad \mu := \begin{pmatrix} 134 \\ 134 \\ 160 \\ 150 \\ 150 \\ 65 \\ 50 \end{pmatrix} \quad \sigma = \begin{pmatrix} 23 \\ 23 \\ 35 \\ 30 \\ 30 \\ 20 \\ 15 \end{pmatrix}$$

$$\mathbf{R} := \begin{pmatrix} 1 & 0.4 & 0.2 & 0.2 & 0.2 & 0 & 0 \\ 0.4 & 1 & 0.4 & 0.2 & 0.2 & 0 & 0 \\ 0.2 & 0.4 & 1 & 0.4 & 0.2 & 0 & 0 \\ 0.2 & 0.2 & 0.4 & 1 & 0.4 & 0 & 0 \\ 0.2 & 0.2 & 0.2 & 0.4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0.4 \\ 0 & 0 & 0 & 0 & 0 & 0.4 & 1 \end{pmatrix}$$

INPUT from standard reliability package e.g. CALREL

- Matrix $\mathbf{U} = [U_i^*]$ – U-space coordinates of Most Probable Failure Points (MPFP) for 5 failure functions.
  $\mathbf{B} = [\beta_i]$

$$
\mathbf{U} = \begin{pmatrix}
-0.939 & -0.459 & -0.753 & -1.383 & -0.737 \\
-0.611 & -0.307 & -1.269 & -1.552 & -1.072 \\
-0.297 & -0.745 & -1.797 & -0.843 & -1.329 \\
-0.781 & -0.546 & -0.715 & 2 \times 10^{-5} & -1.396 \\
-0.569 & -0.224 & 1 \times 10^{-5} & 1 \times 10^{-5} & -1.2 \times 10^{-4} \\
1.21 & 0.914 & 0.458 & 1.219 & 1.194 \\
0 & 0.5 & 1.062 & 0 & 0
\end{pmatrix}
\qquad
\mathbf{B} = \begin{pmatrix} 1.934 \\ 1.514 \\ 2.699 \\ 2.553 \\ 2.614 \end{pmatrix}
$$

- $\mathbf{Y}^* = \mathbf{A}\,\mathbf{U}$, **A** from $\mathbf{R}' = \mathbf{A}\mathbf{A}^T$

3. PRACTICAL CALCULATION OF SENSITIVITY MATRICES

INPUT: $\mathbf{R}'$, **e**, **s**, $[\,\mathbf{Y}^*_{<i>}\,]$, $[\,\beta_{<i>}]$

$$\mathbf{Z} = [\mathbf{Z}^{<i>}] = [\,\mathbf{Y}_i^* / \beta_i\,]$$

AUXILIARY MATRIX

$$\overline{\mathbf{Z}} = \mathbf{R}'^{-1}\mathbf{Z}$$

BIMODAL SUBMATRICES < i&j >

$$\mathbf{W}^{<i\&j>} = \left(\overline{\mathbf{Z}}^{<i>}\,\overline{\mathbf{Z}}^{<j>T}\right) \circ \mathbf{e}$$

UNIMODAL SENSITIVITIES < i >

$$\mathbf{B}_{//R}^{<i>} = -\beta_i \cdot \mathbf{W}^{<i>} \circ \mathbf{s} \quad \Rightarrow \quad \mathbf{P}_{//R}^{<i>} = -\phi(\beta_i) \cdot \mathbf{B}_{//R}^{<i>}$$

BIMODAL SENSITIVITIES < i&j >

$$\mathbf{G}_{//R}^{<i\&j>} = \left[\mathbf{W}^{<i\&j>} + \mathbf{W}^{<i\&j>T} - \gamma_{ij}\left(\mathbf{W}^{<i\&i>} + \mathbf{W}^{<j\&j>}\right)\right] \circ \mathbf{s}$$

$$\mathbf{H}_{//R}^{<i\&j>} = C^{<i\&j>}\mathbf{B}_{//R}^{<j\&j>} + C^{<j\&i>}\mathbf{B}_{//R}^{<j\&j>} + \phi_2\left(\beta_i, \beta_j, \gamma_{ij}\right) \cdot \mathbf{G}_{//R}^{<i\&j>}$$

DITLEVSEN'S UPPER BOUND SENSITIVITY MATRIX

$$\mathbf{P}_{//R}^{BUseries} = \sum_{i=1}^{n_f} \mathbf{P}_{//R}^{<i>} - \sum_{i=2}^{n_f}\left(\mathbf{H}_{//R}^{<i\&j>} \,\Big|\, \max_{j<i} P_{ij}\right)$$

GENERALISED SAFETY INDEX SENSITIVITY MATRIX

$$\mathbf{B}_{//R}^{G} = -\Phi^{-1}\left(\mathbf{P}_{//R}^{BUseries}\right)$$

RESULTS

- Sensitivity matrices $B_{//R}$, $P_{//R}$, $G_{//R}$, $H_{//R}$, $B^{G}_{//R}$, $P^{B}_{//R}$ ($=P^{B}{,_{R}}^{\circ}\, s$) are generated.

(a) Sensitivity matrix of upper Ditlevsen bound for series system (P^B)

$$P^{B}_{//R}\,[\%] = \begin{pmatrix} 0 & -3.187 & -1.684 & -2.627 & -2.066 & 0 & 0 \\ -3.187 & 0 & 1.278 & -0.618 & -1.204 & 0 & 0 \\ -1.684 & 1.278 & 0 & -11.892 & -1.971 & 0 & 0 \\ -2.627 & -0.618 & -11.892 & 0 & -7.383 & 0 & 0 \\ -2.066 & -1.204 & -1.971 & -7.383 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -12.937 \\ 0 & 0 & 0 & 0 & 0 & -12.937 & 0 \end{pmatrix}$$

L-norms are used for measuring of relative change of failure probability

$\Delta P^B/P^B$ due to $\Delta(\rho_{km}+\rho_{mk})$ ie. $s=[2\Delta\rho_{km}/P^B*100]$.

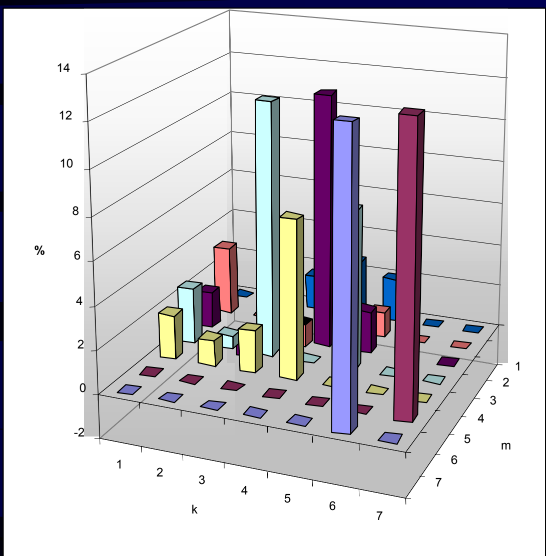
(For no correlation $\Delta\rho_{km}=-2\,\rho_{km}$)

$L_{\infty}(_{P}{}^{B}{}_{//R}) = \max \Delta P^B(-\rho_{km})/P^B$ = 12.9% $\Rightarrow$ the most influential coefficient is ρ_{67}

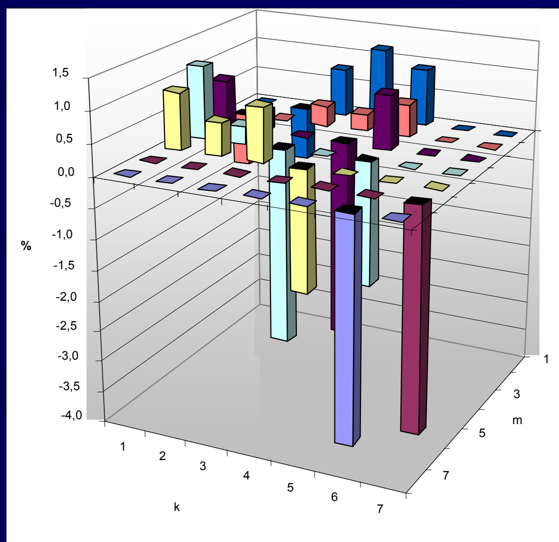
$L_{row}(P^{B}_{//R}) = \max \Delta P^B(-\rho_{xk})/P^B$ = 22.5 % $\Rightarrow$ for the most influential variable x_4;

$L_1(P^{B}_{//R}) = \Delta P^B(-R)/P^B$ = 44.2 % $\Rightarrow$ effect of omitting correlation

RESULTS



$\Delta P^B/P^B$ *for ommiting corr.coeff. (k,m)*



$\Delta P^B/P^B$ *for* $\Delta\rho_{km}$

= deviation from average $\rho=0.29$

RESULTS

(b) Sensitivity of bimodal failure probability

Submatrix $H^{<1\&2>}_{//R}$, for the worst combination of failure surfaces g_1 & g_2 is composed using normalization matrix $s=[2\,\Delta\rho_{km} / P_{12} *100\%]$.

When $\Delta\rho_{km} = -2\rho_{km}$ (correlation omitted) it shows high influence of correlation coefficients $\rho_{54}+\rho_{45}$ and $\rho_{67}+\rho_{76}$ on P_{12} (→should be analysed for real systems - parallel, series):

$$H^{<1\&2>}_{//R}\,[\%] = \begin{pmatrix} 0 & -6.827 & -3.503 & -7.467 & -5.894 & 0 & 0 \\ -6.827 & 0 & -8.643 & -6.404 & -4.461 & 0 & 0 \\ -3.503 & -8.643 & 0 & -6.63 & -4.492 & 0 & 0 \\ -7.467 & -6.404 & -6.63 & 0 & -19.308 & 0 & 0 \\ -5.894 & -4.461 & -4.492 & -19.308 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -14.663 \\ 0 & 0 & 0 & 0 & 0 & -14.663 & 0 \end{pmatrix}$$

$L_\infty(H^{<1\&2>}_{//R}) = \max \Delta P_{12}(\bullet)/P_{12} = 19.3\%$ ⇒ for the most influential coefficient ρ_{45}

$L_{row}(H^{<1\&2>}_{//R}) = \max\Delta P_{12}(\bullet)/P_{12} = 39.81\%$ ⇒ for the most influential variable x_4;

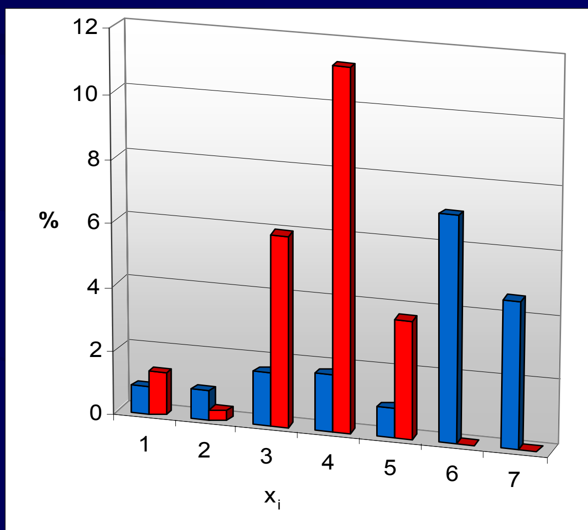
$L_1(H^{<1\&2>}_{//R}) = \Delta P_{12}(-R)/P_{12} = 88.3\%$ ⇒ effect of neglecting correlation

ACCURACY

❑ **Sensitivity estimates are extensively compared to FDM (using CALREL runs). Brief comparison, within design oriented FORM concept, is given in Table:**

P^{B0}(**R**)= 0.072	1	2	3	4
R(ρ_{km}+ $\Delta\rho_{km}$)	$P^B(\bullet)$ estimate	$P^B(\bullet)$ CALREL	error [%] (1-2) / 1	error [%] (1-2) / P^{B0}
$\Delta\rho_{km}=20\%\,\rho_{km}$	0,0790	0,0782	1,01	1,1
$\Delta\rho_{km}$ is deviation from average ρ	0,0690	0,0688	0,29	0,3
$\Delta\rho_{km} = -\rho_{km}$ totally omitting correlation	0,0400	0,0360	10,00	5,55

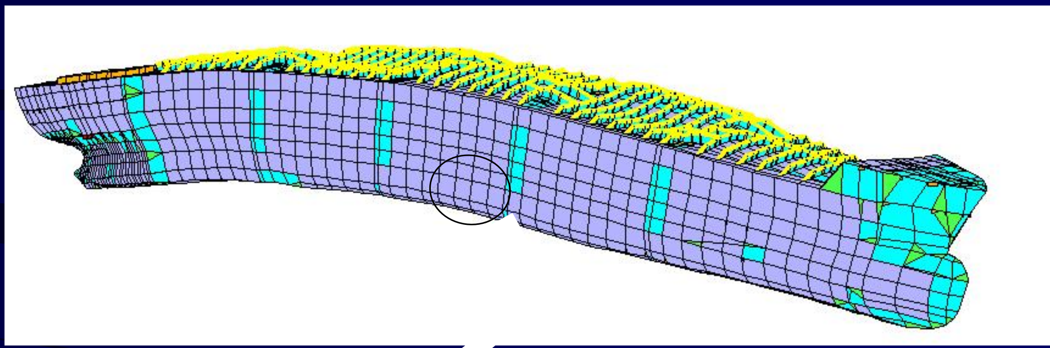
❑ ***Comparison of influence: $\Delta P^B/P^B$ for taking change in st.dev. of $\Delta\sigma$=20% and $\Delta P^B/P^B$ for taking variable x_1 as uncorrelated***

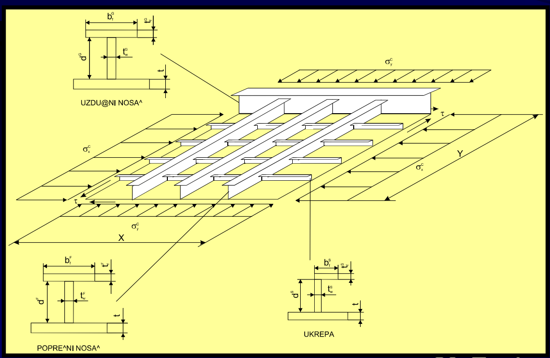


EXAMPLE 2 PRACTICAL APPLICATION:

Oil Product Tanker 65,200 dwt, DnV

L_{PP} = 175.5 m
B = 40 m
T = 13 m
D = 17.9 m
V = 16.9 kn





Typical stiffened panel macroelement

EXAMPLE 2 PROBLEM DEFINITION :

- **random variables $x = \{x_i\} = \{sig_x, sig_y, tau_{xy}, p, t_p, E, sig_{Yield}\}$**
- **marginal distributions: Normal (x_1-x_3, $x_{5\text{-}7}$) and extreme (x_4);**
- **prescribed means μ, standard deviations σ and correlation matrix R;**
- **series system; Class. soc.(CRS) failure surfaces (yield, buckling) $g_j < 0$; (j=1- 5)**

Limit state functions - CRS criteria :

PCLB - Panel collapse local buckling (x)
PCTB - Panel collapse transverse buckling (y)
PCMY - Panel collapse membrane yield
SYCF - Stiffener yield compression flange
SYCP - Stiffener yield compression plate

$$\mu := \begin{pmatrix} -120 \\ -70 \\ 48 \\ -150 \\ 15.5 \\ 210000 \\ 235 \end{pmatrix} \qquad \sigma := \begin{pmatrix} 18 \\ 11.5 \\ 7.2 \\ 22.5 \\ 0.3 \\ 4200 \\ 4.7 \end{pmatrix}$$

$$R = \begin{pmatrix} 1 & 0.5 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0.5 & 1 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0.5 & 0.5 & 1 & 0.5 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0.5 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 & 1 \end{pmatrix}$$

INPUT

- Matrix **U** = $[U_i^*]$ – U-space coordinates of MPFP points for 5 failure functions.

$$\mathbf{B} = \begin{pmatrix} 3.171 \\ 8.042 \\ 4.583 \\ 4.665 \\ 2.963 \end{pmatrix} \begin{matrix} \text{PCLB} \\ \text{PCTB} \\ \text{PCMY} \\ \text{SYCF} \\ \text{SYCP} \end{matrix}$$

U :=

	PCLB	PCTB	PCMY	SYCF	SYCP
SIG_X	−3.03	−5.726	−4.419	−4.479	−2.864
SIG_Y	−0.5519	−5.114	−0.9607	−0.5179	−0.4616
TAU_{XY}	−0.08912	0.	0.555	0.1539	−0.0266
PRESS	−0.3452	0.	−0.4996	0.5961	−0.1031
t_P	−0.3471	−1.25	0.	−0.5147	−0.2966
E	−0.1957	−0.705	0.	−0.2972	−0.1713
SIG_{YIELD}	−0.5324	−1.917	0.	−0.8405	−0.4844

- **Matrices B, P, G, H, Z, W are (re)calculated from U:**

P_{13}= 2.29*10^{-6}, etc.

P^B= 0.00153, β_G=2.962

RESULTS

- **Sensitivity matrices $B_{//R}$, $P_{//R}$, $G_{//R}$, $H_{//R}$, $B^G_{//R}$, $P^B_{//R}$ (=$P^B{}_{,R}\circ$ s) are generated.**

(a) Sensitivity matrix for generalised safety index of series system (B^G)

$B^G_{//R}$ [%] =

0	−10.696	0.027	−5.035	0	0	0
−10.696	0	8.115×10^{-3}	−0.967	0	0	0
0.027	8.115×10^{-3}	0	4.341×10^{-3}	0	0	0
−5.035	−0.967	4.341×10^{-3}	0	0	0	0
0	0	0	0	0	-4.053×10^{-4}	−0.03
0	0	0	0	-4.053×10^{-4}	0	−0.016
0	0	0	0	−0.03	−0.016	0

L-norms used for measuring of relative change of safety index β_G :

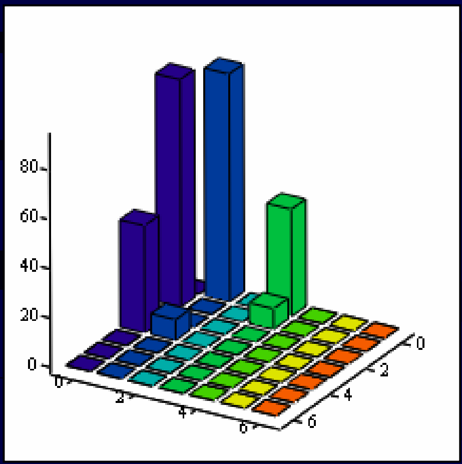
$\Delta B^G/\beta_G$ due to $\Delta(\rho_{km}+\rho_{mk})$ ie. s=[2$\Delta\rho_{km}$ /β_G*100].

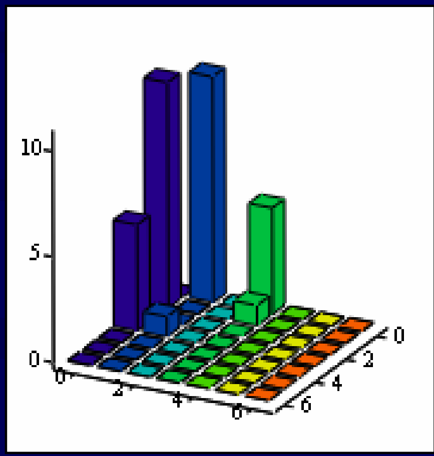
$L_\infty(B^G_{//R})$ = max $\Delta B^G(\rho_{km})/\beta_G$ = -10.70 % $\Rightarrow$ the most influential coefficient is ρ_{12}

L_{row} ($B^G_{//R}$) =max $\Delta B^G(\rho_{xk})$ /β_G = -15.70 % $\Rightarrow$ for the most influential variable x_1;

L_1 ($B^G_{//R}$) = ΔB^G(R) /β_G = - 33.41 % $\Rightarrow$ for ρ = 0. 95 correlated case

RESULTS





$\Delta P^B/P^B$ for ρ = 0. 95 correlated case -($\Delta B^G/\beta_G$) ***for ρ = 0. 95 correlated case***

RESULTS

(b) Sensitivity of bimodal failure probability

Submatrix $H^{<1\&5>}{}_{//R}$, for the combination of failure surfaces PCLB & SYCP is composed using normalization matrix $s = [2\ \Delta\rho_{km} / P_{12} * 100\%\ ]$.

$H^{<1\&5>}{}_{//R}$ [%] =

0	61.675	-1.578×10^{-3}	54.701	0	0	0
61.675	0	-3.025×10^{-4}	10.483	0	0	0
-1.578×10^{-3}	-3.025×10^{-4}	0	-2.825×10^{-4}	0	0	0
54.701	10.483	-2.825×10^{-4}	0	0	0	0
0	0	0	0	0	6.499×10^{-3}	0.53
0	0	0	0	6.499×10^{-3}	0	0.273
0	0	0	0	0.53	0.273	0

%

$L_\infty(H^{<1\&5>}{}_{//R})$ = max $\Delta P_{12}(\bullet)/P_{12}$ = 61.67% $\Rightarrow$ **for the most influential coefficient ρ_{12}**

$L_{row}(H^{<1\&5>}{}_{//R})$ = max$\Delta P_{12}(\bullet)/P_{12}$ = 116.37 % $\Rightarrow$ **for the most influential variable x_1;**

ACCURACY

- Sensitivity estimates are extensively compared to FDM (using CALREL runs). Brief comparison, within design oriented FORM concept, is given in Table:

β_G= 2.965	1	2	3	4
$R(\rho_{km})$ = $R(\rho_{km0} + \Delta\rho_{km})$	$\beta(\cdot)$ estimate	$\beta(\cdot)$ CALREL	error [%] $\frac{(1-2)}{1}$	error [%] $\frac{(1-2)}{\beta_G}$
$\Delta\rho_{km}$= 20% ρ_{km} ρ_{km}= 0,6	2,875	2,912	1,28	1,24
$\Delta\rho_{km}$= 0.95-ρ_{km} , ρ_{km}= 0,95	2,675	2,754	2,95	2,66
$\Delta\rho_{km}$ = -60% ρ_{km} ρ_{km}= 0,2	4,001	3,118	22,1	29,78

4. CONCLUSIONS

- The comprehensive and numerically efficient method for sensitivity analysis in FORM is presented. It does not require either the derivatives of the transformation matrices or their recalculation.

- It enables direct calculation of sensitivity matrices for componential and system reliability measures with respect to all correlation coefficients at once.

- The sensitivity matrices w.r.t. correlation coefficients or their parameters are available as the intermediate results of the failure probability calculation.

CONCLUSIONS (cont.)

- These matrices jointly used, enable **efficient identification of most significant correlation related parameters** in reliability analysis.

- Comparison of sensitivity estimation with FDM approach for system and component reliability measures proves to be **sufficiently accurate for design purposes**.

- The presented method can be **easily implemented in existing procedures** and computer codes for reliability analysis. It does not require additional structural response evaluation

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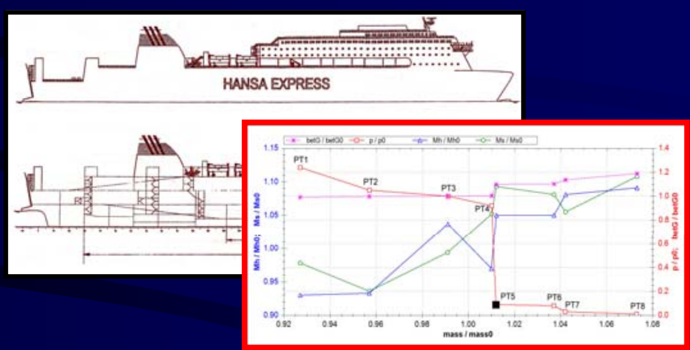

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